Mathematical Model

The control systems can be represented with a set of mathematical equations known as **mathematical model**. These models are useful for analysis and design of control systems.

Analysis of control system means finding the output when we know the input and mathematical model.

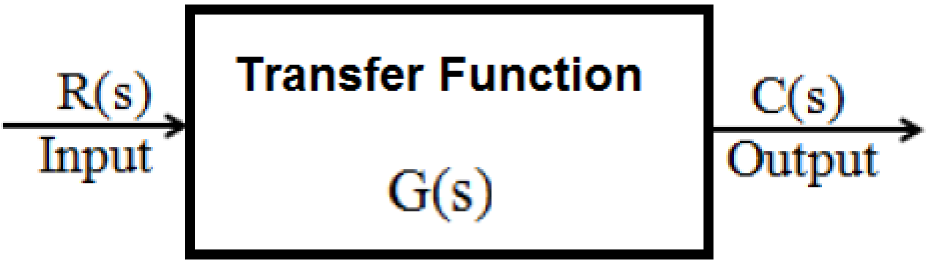
Design of control system means finding the mathematical model when we know the input and the output.

The following mathematical models are mostly used.

- ☒ Differential equation model
- ☒ Transfer function model
- ☒ State space model

Transfer Function Representation

A **transfer function** represents the relationship between the output signal of a control system and the input signal, for all possible input values. A block diagram is a visualization of the control system which uses blocks to represent the transfer function, and arrows which represent the various input and output signals.



In a Laplace Transform,

if the input is represented by R(s) and the output is represented by C(s), then the transfer function will be:

$$G(s) = \frac{C(s)}{R(s)}$$

That is,

the transfer function of the system multiplied by the input function gives the output function of the system.

$$C(s) = R(s) * G(s)$$

Procedure for determining the transfer function of a control system are as follows:

1. We form the equations for the system.
2. Take Laplace transform of the system equations, assuming initial conditions as zero.
3. Specify system output and input.

Lastly we take the ratio of the Laplace transform of the output and the Laplace transform of the input which is the required transfer function

$$\text{Transfer function} = G(s) = \left.\frac{\mathcal{L}[\text{output}]}{\mathcal{L}[\text{input}]}\right|_{\text{zero initial conditions}}$$

$$= \frac{C(s)}{R(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1}s + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n}$$

$$G(s) = \frac{k\,(s-z_1)(s-z_2)\ldots\ldots\ldots\ldots\ldots(s-z_m)}{(s-p_1)(s-p_2)\ldots\ldots\ldots\ldots\ldots(s-p_n)}$$

$$m \leq n$$

Where: *$z_1, z_2, \ldots\ldots, z_m$ are called the zeros of the system, or zeros of G(s), because at $s=z_1$ or $s=z_2$ or …….$s=z_m$ the amplitude of G(s)=0.*

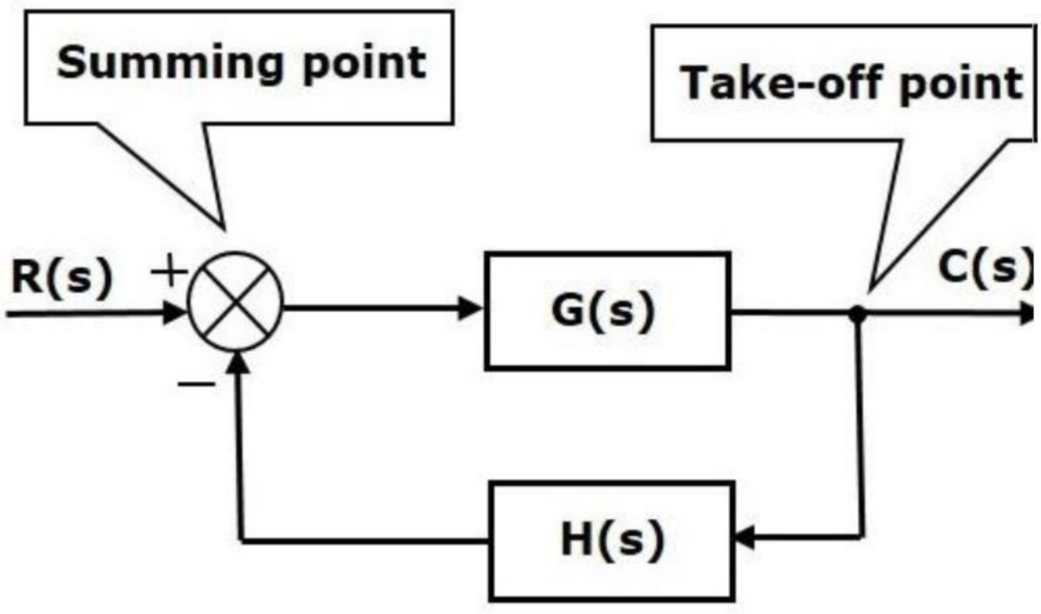
$p_1, p_2, \ldots\ldots, p_n$ are called the poles of the system, or poles of G(s), because at $s=p_1$ or $s=p_2$ or …….$s=p_n$ the amplitude of G(s) =Infinity.

Block Diagrams

Block diagrams consist of a single block or a combination of blocks. These are used to represent the control systems in pictorial form.

Basic Elements of Block Diagram

The basic elements of a block diagram are **a block**, **the summing point** and the **take-off point.** The following consider the block diagram of a closed loop control system as shown in the following figure to identify these elements.

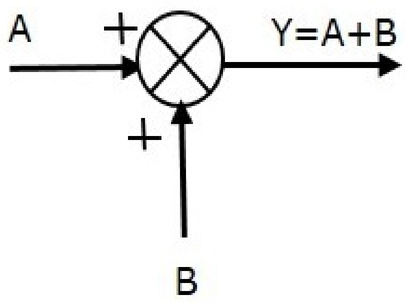


The block diagram above consists of two blocks having transfer functions G(s) and H(s). It is also having one summing point and one take-off point. Arrows indicate the direction of the flow of signals.

Summing Point

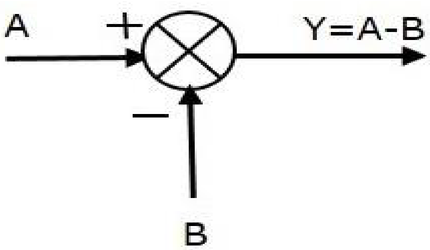
The summing point is represented with a circle having cross (X) inside it. It has two or more inputs and single output. It produces the summation or subtraction or combination of summation and subtraction of the inputs based on the polarity of the inputs.

- **The following figure shows the summing point with two inputs** (A, B) and one output (Y). Here, the inputs A and B have a positive sign. So, the summing point produces the output, Y as **sum of A and B**



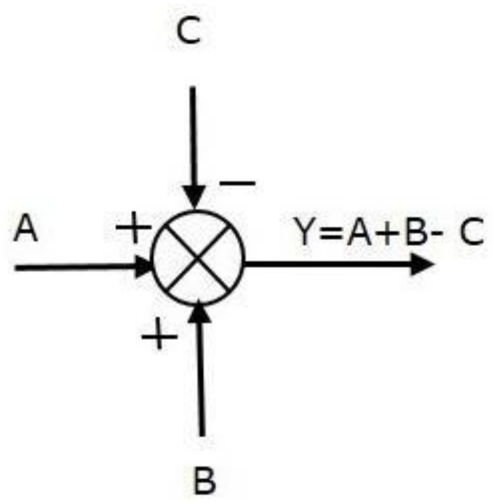
The following figure shows the summing point with two inputs (A, B) and one output (Y). Here, the inputs A and B are having opposite signs, A is having positive sign and B is having negative sign. the summing point produces the output **Y** as the **difference of A and B**

Y = A + (-B) = A - B.



❖ **The following figure shows the summing point with three inputs.** (A, B, C) and one output (Y). Here, the inputs A and B are having positive signs and C is having a negative sign. So, the summing point produces the output **Y** as

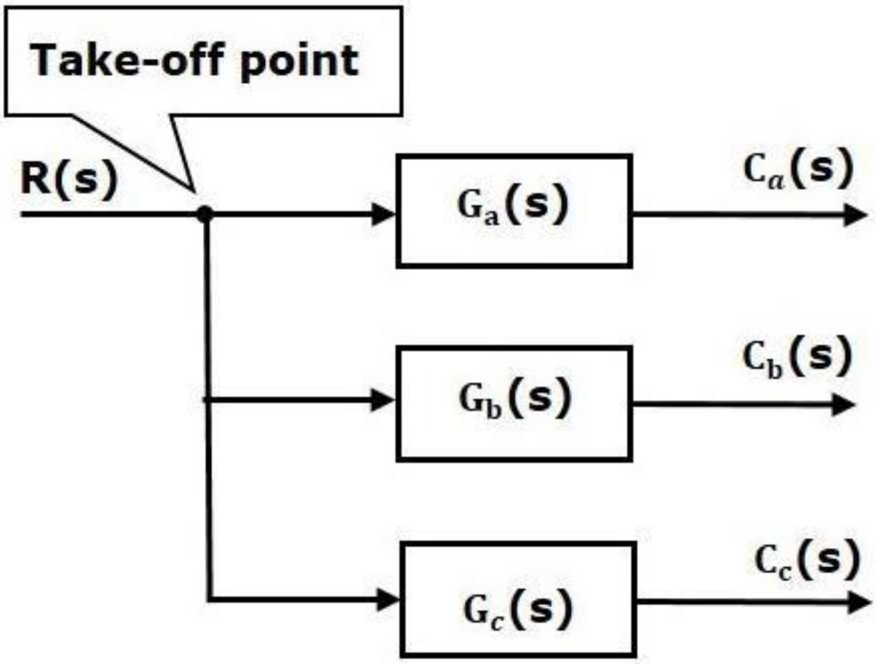
$$Y = A + B + (-C) = A + B - C.$$



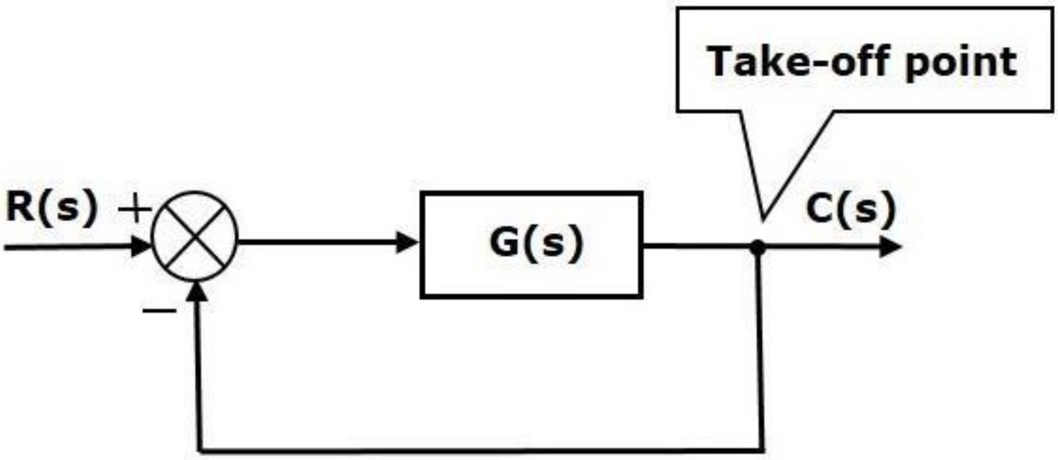
Take-off Point

The take-off point is a point from which the same input signal can be passed through more than one branch.

That means with the help of take-off point, we can apply the same input to one or more blocks, summing points. In the following figure, the take-off point is used to connect the same input, R(s) to two more blocks.



In the following figure, the take-off point is used to connect the output C(s), as one of the inputs to the summing point.



Answer the following

1. Which terminology deals with the excitation or stimulus applied to the system from an external source for the generation of an output?

a. Input signal
b. Output signal
c. Error signal
d. Feedback signal

2. Which among the following is not an advantage of an open loop system?

a. Simplicity in construction & design
b. Easy maintenance
c. Rare problems of stability
d. Requirement of system recalibration from time to time

3. The output signal is fed back at the input side from the _________point

a. Summing
b. Differential
c. Take-off
d. All of the above

4.Define the transfer function

5.What is meaning of the block diagram, explain its elements.

6. What is the meaning of analysis of control system?

7. What is the meaning of design of control system?