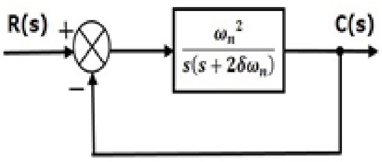
Time Response of Second Order Control System

The order of a control system is determined by the power of ***'s'*** in the denominator of its transfer function. If the power of s in the denominator of the transfer function of a control system is 2, then the system is said to be **second order control system**. The general expression of the transfer function of a second order control system is given as



$$G(S) = \frac{\omega_n^2}{s(s+2\xi\omega_n)}$$

H(S) =1

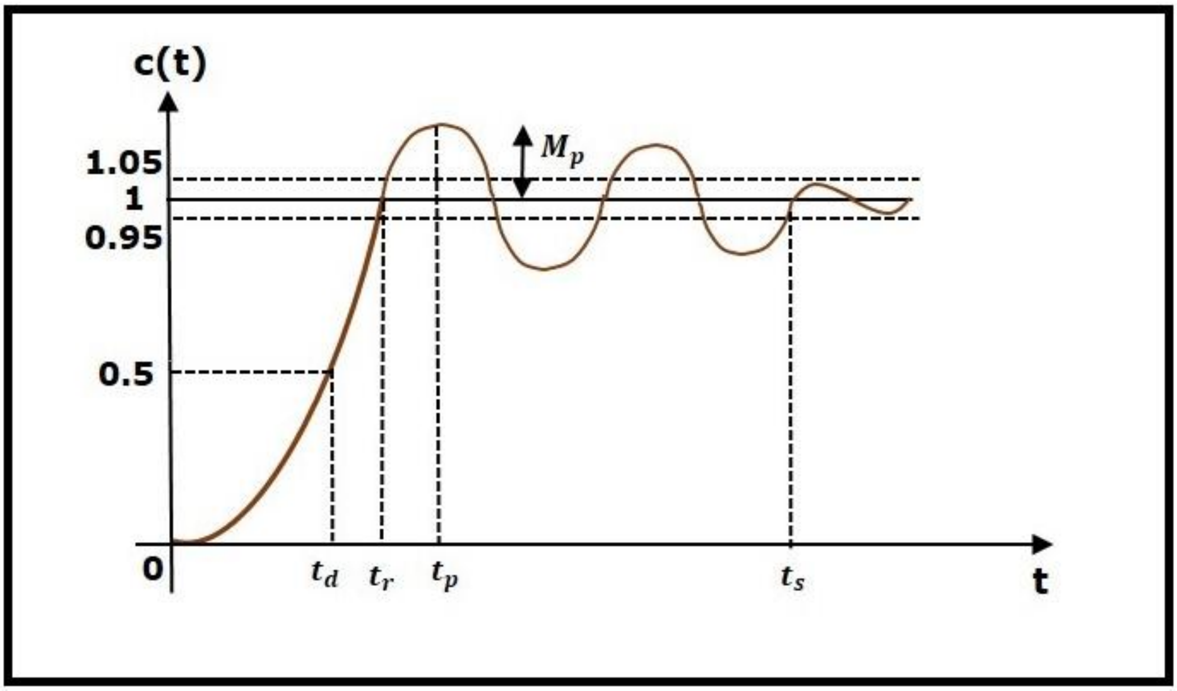
$$\frac{C(s)}{R(s)} = \frac{G(S)}{1+G(S)H(S)}$$

$$\frac{C(S)}{R(S)} = \frac{\omega_n^2/s(s+2\xi\omega_n)}{1+\frac{\omega_n^2}{s(s+2\xi\omega_n)}}$$

$$\boxed{\frac{C(S)}{R(S)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}}$$ eq.1

Transient response specification of second order system

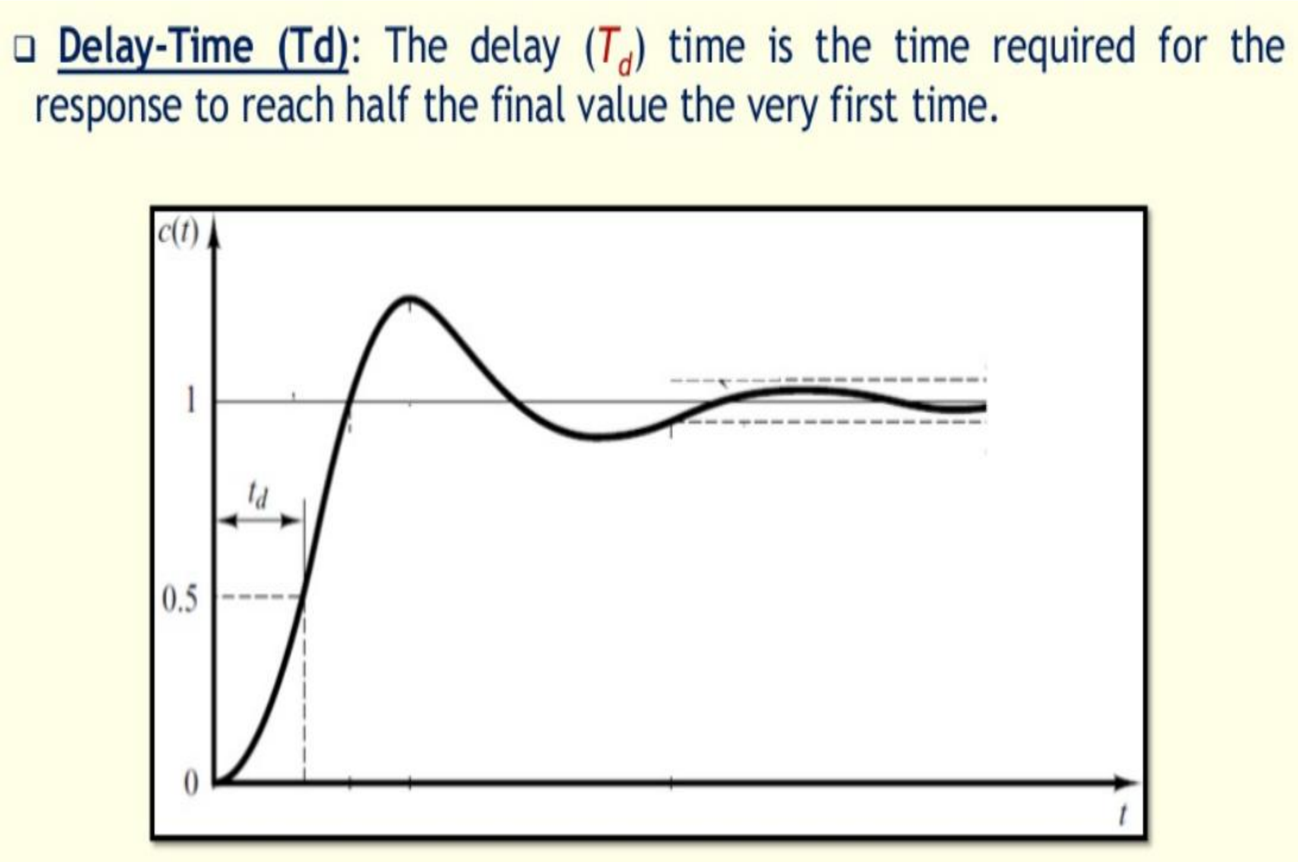
The performance of the control system are expressed in terms of transient response to a unit step input because it is easy to generate initial condition basically are zero.



Following are the common transient response characteristics:

1. <u>Delay Time.</u>

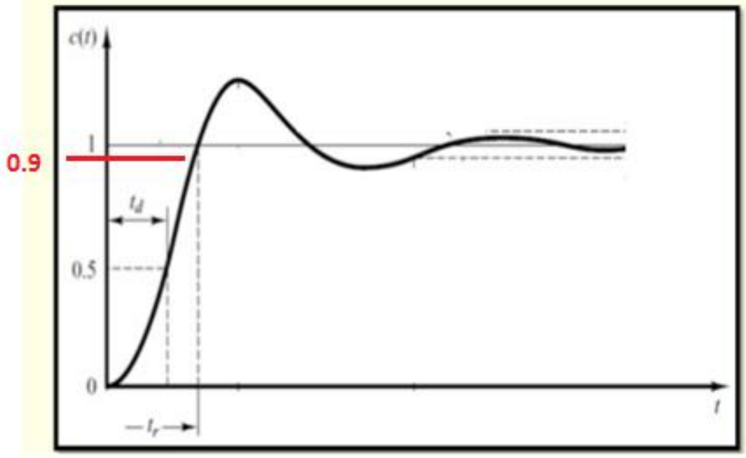
The time required for the response to reach ***50%*** of the final value in the first time is called the delay time.



2. Rise Time.

The time required for response to rising from 10% to 90% of final value, for an overdamped system and 0 to 100% for an underdamped system is called the rise time of the system.

- 10% to 90% of its final value, ➔ over damped systems
- 5% to 95% of its final value, ➔ Critical damped systems
- or 0% to 100% of its final value. ➔ under damped systems



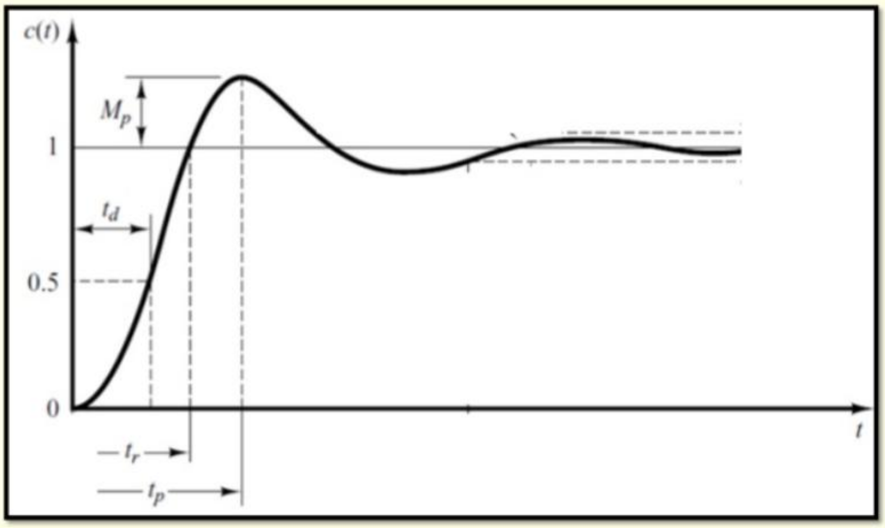
$$t_r = \frac{\pi - \theta}{\omega_d}$$

$$\theta = \tan^{-1}\left(\frac{\omega_n\sqrt{1-\zeta^2}}{\zeta\omega_n}\right)$$

$$t_r = \frac{\pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}}{\omega_n\sqrt{1-\xi^2}}$$

3. Peak Time.

- **Peak Time (Tp)**: The peak time is the time required for the response to reach the first (maximum) peak of the overshoot.

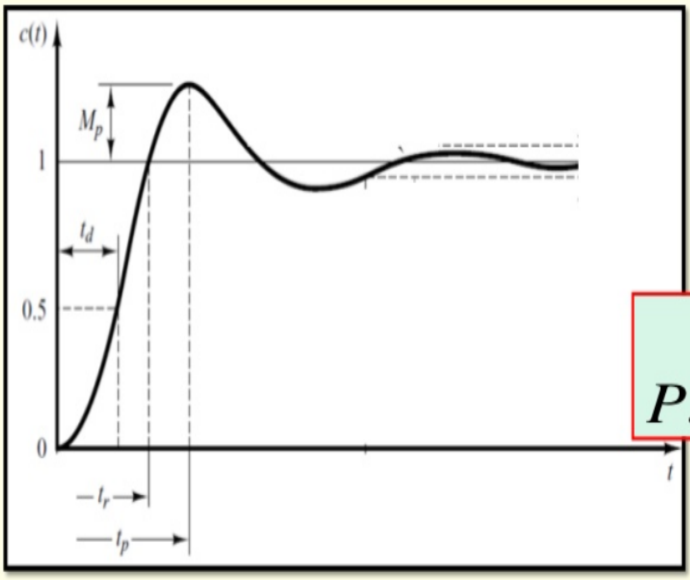


$$t_p = \frac{\pi}{\omega_d}$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

4. Maximum Peak.

The difference between the peak of 1st time and steady output is called the maximum overshoot. It is defined by



$$MP = e^{-(\frac{\zeta}{\sqrt{1-\zeta^2}})\pi}$$

$$P.O = \frac{Mp}{final\ value} x100\%$$

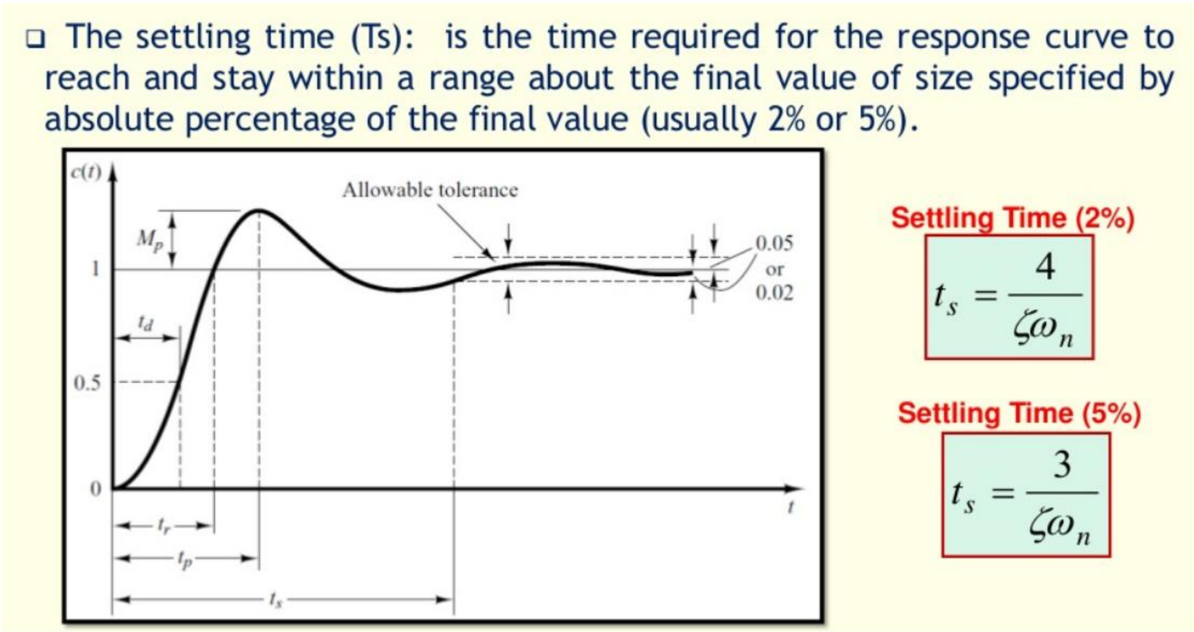
$$P.O = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} x100\%$$

$$\text{Maximum percent overshoot} = \frac{c(t_p)-c(\infty)}{c(\infty)}$$

$$\%M_P = e^{\frac{\pi\xi}{\sqrt{1-\xi^2}}} \times 100$$

5. **Settling Time**.

The time that is required for the response to reach and stay within the specified range (2% to 5%) of its final value is called the settling time.



$$t_s = \frac{4}{\xi\omega_n}$$

6. <u>Steady State error.</u>

The difference between actual output and desired output as time 't' tends to infinity is called the steady state error of the system.

$$e_{ss} = \lim_{t\to\infty}[r(t) - C(t)]$$

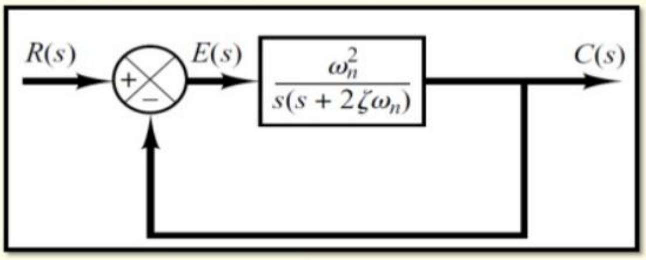
- $\zeta \rightarrow$ damping ratio of the second order system, which is a measure of the degree of resistance to change in the system output.
- $\omega_n \rightarrow$ un-damped natural frequency of the second order system, which is the frequency of oscillation of the system without damping

- According the value of ζ , a second-order system can be set into one of the four categories:
  - Case 1: Over damped response ($\xi > 1$)
  - Case 2: Critically damped response ($\xi = 1$)
  - Case 3: Under damped response ($0 < \xi < 1$)
  - Case 4: No damped response ($\xi = 0$)

Example 1

- Consider the system shown in following figure, where damping ratio is 0.6 and natural undamped frequency is 5 rad/sec.

Obtain the rise time t_r, peak time t_p, maximum overshoot M_p, and settling time 2% and 5% criterion t_s
when the system is subjected to a unit-step input.



Solution:

$\xi = 0.6$ $\omega n = 5$ rad/sec

1. Rise Time:

$$t_r = \frac{\pi - \theta}{\omega_d}$$

$$t_r = \frac{3.141 - \theta}{\omega_n \sqrt{1-\zeta^2}} \rightarrow \theta = \tan^{-1}\left(\frac{\omega_n \sqrt{1-\zeta^2}}{\zeta \omega_n}\right) = 0.93 \text{ rad}$$

$$t_r = \frac{3.141 - 0.93}{5\sqrt{1-0.6^2}} = 0.55s$$

2. Peak Time:

$$t_p = \frac{\pi}{\omega_d} \rightarrow t_p = \frac{3.141}{4} = 0.785s$$

3. Settling Time (2%):

$$t_s = \frac{4}{\zeta\omega_n} \rightarrow t_s = \frac{4}{0.6\times 5} = 1.33s$$

Settling Time (5%):

$$t_s = \frac{3}{\zeta\omega_n} \rightarrow t_s = \frac{3}{0.6\times 5} = 1s$$

4. Maximum Overshoot:

$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} = e^{-\frac{3.141\times 0.6}{\sqrt{1-0.6^2}}} = 0.095$$

$$P.O = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100 = 9.5\%$$