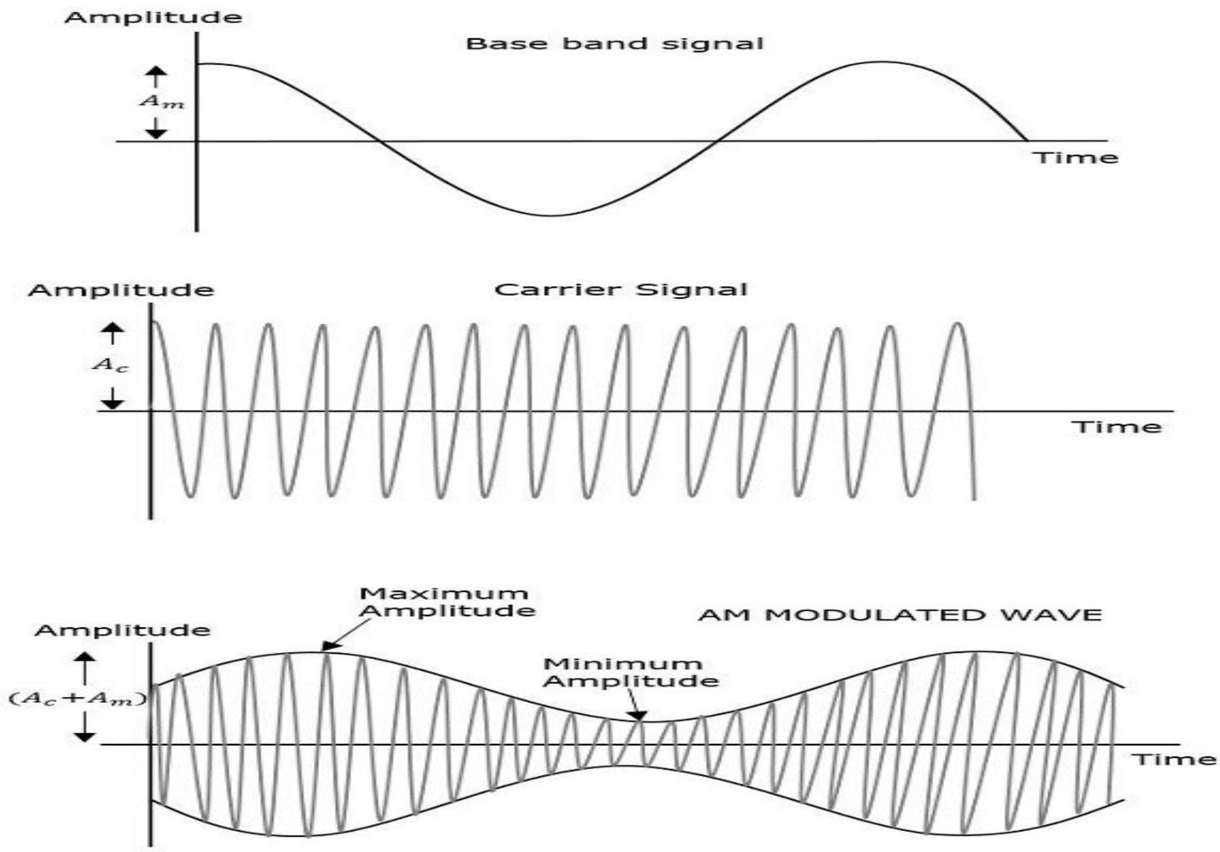
Mathematical Expressions of Amplitude Modulation

According to the standard definition, "The amplitude of the carrier signal varies in accordance with the instantaneous amplitude of the modulating signal." Which means, the amplitude of the carrier signal containing no information varies as per the amplitude of the signal containing information, at each instant. This can be well explained by the following figures.



The first figure shows the modulating wave, which is the message signal. The next one is the carrier wave, which is a high frequency signal and contains no information. While, the last one is the resultant modulated wave

Mathematical Expressions

Following are the mathematical expressions for these waves.

Time-domain Representation of the Waves

Let the modulating signal be

1. The Modulating Signal

$$m(t) = A_m \cos(2\pi f_m t)$$

A_m : the amplitude of the modulating signal

f_m : the frequency of the modulating signal

2. The Carrier Signal

$$c(t) = A_c \cos(2\pi f_c t)$$

A_c : the amplitude of the carrier signal

f_c : the frequency of the carrier signal

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3. AM Modulation Process

the equation of Amplitude Modulated wave will be

$$s(t) = [A_c + A_m \cos(2\pi f_m t)] \cos(2\pi f_c t)$$ (Equation 1)

4. Modulation Index

A carrier wave, after being modulated, if the modulated level is calculated, then such an attempt is called as **Modulation Index** or **Modulation Depth**. It states the level of modulation that a carrier wave undergoes.

Rearrange the Equation 1 as below.

$$s(t) = A_c \left[1 + \left(\frac{A_m}{A_c}\right) \cos(2\pi f_m t)\right] \cos(2\pi f_c t)$$

$$s(t) = A_c [1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t)$$ (Equation 2)

Where,

μ is Modulation index and it is equal to the ratio of A_m and A_c

Mathematically, we can write it as

$$\mu = \frac{A_m}{A_c}$$ (Equation 3)

Hence, we can calculate the value of modulation index by using the above formula, when the amplitudes of the message and carrier signals are known. Now, let us derive one more formula for Modulation index by considering Equation 1. We can use this formula for calculating modulation index value, when the maximum and minimum amplitudes of the modulated wave are known

Let

A_{max} : is the maximum amplitudes of the modulated wave.

A_{min} : is the minimum amplitudes of the modulated wave.

We will get the maximum amplitude of the modulated wave, when $\cos(2\pi f_m t)$ is 1.

$$\Rightarrow \boxed{A_{max} = A_c + A_m} \qquad \text{(Equation 4)}$$

We will get the minimum amplitude of the modulated wave, when $\cos(2\pi f_m t)$ is -1.

$$\Rightarrow \boxed{A_{min} = A_c - A_m} \qquad \text{(Equation 5)}$$

Add Equation 4 and Equation 5.

$$A_{\max} + A_{\min} = A_c + A_m + A_c - A_m = 2A_c$$

$$\Rightarrow \quad \boxed{A_c = \frac{A_{\max} + A_{\min}}{2}} \qquad \text{(Equation 6)}$$

Subtract Equation 5 from Equation 4.

$$A_{\max} - A_{\min} = A_c + A_m - (A_c - A_m) = 2A_m$$

$$\Rightarrow \quad \boxed{A_m = \frac{A_{\max} - A_{\min}}{2}} \qquad \text{(Equation 7)}$$

The ratio of Equation 7 and Equation 6 will be as follows.

$$\mu = \frac{A_m}{A_c} = \frac{(A_{max} - A_{min})/2}{(A_{max} + A_{min})/2}$$

$$\Rightarrow \quad \boxed{\mu = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}} \qquad \text{(Equation 8)}$$

Example 1

A modulating signal $m(t) = 10\cos(2\pi \times 10^3 t)$ **is amplitude modulated with a carrier signal** $c(t) = 50\cos(2\pi \times 10^5 t)$ **. Find,**

1. **The modulation index**
2. **The carrier power**
3. **The power required for transmitting AM wave**

Solution

Step 1

Given, the equation of modulating signal as

$$m(t) = 10\cos(2\pi \times 10^3 t)$$

We know the standard equation of modulating signal as

$$m(t) = A_m \cos(2\pi f_m t)$$

By comparing the above two equations, we will get

Amplitude of modulating signal as $A_m = 10volts$

and Frequency of modulating signal as $f_m = 10^3 Hz = 1KHz$

Step 2

Given, the equation of carrier signal is

$$c(t) = 50\cos\left(2\pi \times 10^5 t\right)$$

The standard equation of carrier signal is

$$c(t) = A_c \cos(2\pi f_c t)$$

By comparing these two equations, we will get

Amplitude of carrier signal as $A_c = 50 volts$

and Frequency of carrier signal as $f_c = 10^5 Hz = 100KHz$

1. The Modulation index

$$\mu = \frac{A_m}{A_c}$$

Substitute, A_m and A_c values in the above formula.

$$\mu = \frac{10}{50} = 0.2$$

Therefore, the value of **modulation index is 0.2** and percentage of modulation is 20%.

2. The carrier power

The formula for Carrier power, $P_c =$ is

$$P_c = \frac{{A_c}^2}{2R}$$

Assume $R = 1\Omega$ and substitute A_c value in the above formula.

$$P_c = \frac{(50)^2}{2(1)} = 1250W$$

Therefore, the **Carrier power**, P_c is **1250 watts**.

3. the power required for transmitting AM wave

the formula for **power** required for **transmitting AM** wave is

$$P_t = P_c\left(1 + \frac{\mu^2}{2}\right)$$

Substitute P_c and μ values in the above formula.

$$P_t = 1250\left(1 + \frac{(0.2)^2}{2}\right) = 1275W$$

Therefore, the **power required for transmitting AM** wave is **1275 watts**.

Example 2

The equation of amplitude wave is given by $s(t) = 20\left[1 + 0.8\cos\left(2\pi \times 10^3 t\right)\right]\cos\left(4\pi \times 10^5 t\right)$

Find

1. Carrier Power
2. Sideband Power
3. The bandwith of AM wave

Solution

Given, the equation of Amplitude modulated wave is

$$s\left(t\right) = 20\left[1 + 0.8\cos\left(2\pi \times 10^3 t\right)\right]\cos\left(4\pi \times 10^5 t\right)$$

Re-write the above equation as

$$s\left(t\right) = 20\left[1 + 0.8\cos\left(2\pi \times 10^3 t\right)\right]\cos\left(2\pi \times 2 \times 10^5 t\right)$$

We know the equation of Amplitude modulated wave is

$$s\left(t\right) = A_c\left[1 + \mu\cos(2\pi f_m t)\right]\cos(2\pi f_c t)$$

By comparing the above two equations, we will get

Amplitude of carrier signal as $A_c = 20volts$

Modulation index as $\mu = 0.8$

Frequency of modulating signal as $f_m = 10^3 Hz = 1KHz$

Frequency of carrier signal as $f_c = 2 \times 10^5 Hz = 200KHz$

1. Carrier Power

The formula for Carrier power, P_c is

$$P_c = \frac{{A_c}^2}{2R}$$

Assume $R = 1\Omega$ and

substitute A_c value in the above formula.

$$P_c = \frac{(20)^2}{2\,(1)} = 200W$$

Therefore, the **Carrier power**, $P_c =$ **200 watts**.

2. Side band Power

We know the formula for total side band power is

$$P_{SB} = \frac{P_c \mu^2}{2}$$

Substitute P_c and μ values in the above formula.

$$P_{SB} = \frac{200 \times (0.8)^2}{2} = 64W$$

Therefore, the **total side band power** is **64 watts.**

3. the Bandwith of AM wave

We know the formula for bandwidth of AM wave is

$$BW = 2f_m$$

Substitute f_m value in the above formula.

$$BW = 2\,(1K) = 2KHz$$

Therefore, the **bandwidth** of AM wave is **2 KHz.**