Al-Rasheed University College

Dept. of Computer Eng. Techniques

Digital Logic – First Year

References

1. Digital Design — By: Morris Mano
2. Digital Fundamentals — By: F. Floyd

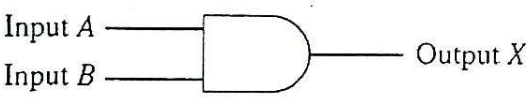
Sheet No. 1 (العملي)

1. Logic Gates

A logic gate is a basic building block of a digital circuit that has two inputs and one output. The relationship between the i/p and the o/p is based on a certain logic. These gates are implemented using electronic switches like transistors, diodes. But, in practice basic logic gates are built using CMOS technology, FETS and MOSFET (Metal Oxide Semiconductor FET)s. Logic gates are used in microprocessors, microcontrollers, embedded system applications and in electronic and electrical project circuits. The basic logic gates are categorized into seven: AND, OR, XOR, NAND, NOR, XNOR and NOT.

1.1. AND Gate

The AND gate is an electronic circuit that gives a **high** output (1) only if **all** its inputs are high.



Symbol

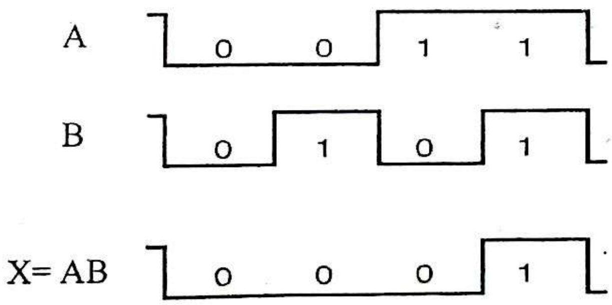
Truth Table for a Two-Input AND Gate

Input		**Output**
A	**B**	**X= AB**
0	0	0
0	1	0
1	0	0
1	1	1

The algebraic expression of the logical AND gate.

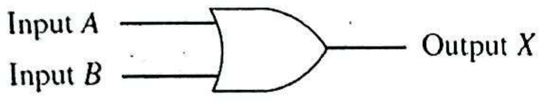
$$X = A \cdot B = AB$$

Figure below shows timing diagram for AND gate with A = $(1100)_2$, B = $(1010)_2$



1.2.OR Gate

The OR gate is an electronic circuit that gives a high output (1) if **one or more** of its inputs are high. A plus (+) is used to show the OR operation.



Symbol

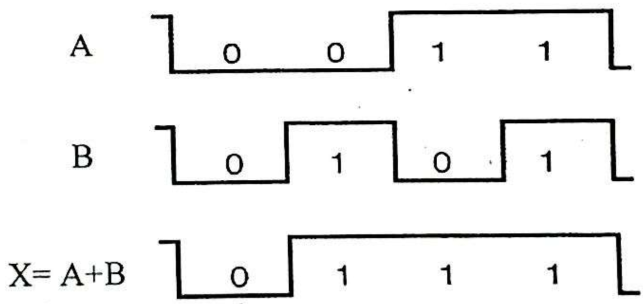
Truth Table for a Two-Input OR Gate

Input		Output
A	**B**	**X= A + B**
0	0	0
0	1	1
1	0	1
1	1	1

ie algebraic expression of the logical OR gate.

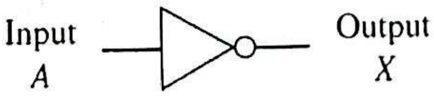
$$X = A + B$$

Figure below shows timing diagram for OR gate with A = $(1100)_2$, B = $(1010)_2$



1.3.NOT Gate (Inverter)

The inverter is used to complement, or invert, a digital signal. It has a single input and a single output. If a HIGH level (1) comes in, it produces a Low-level (0) output. If a LOW level (0) comes in, it produces a High-level (1) output.



Symbol

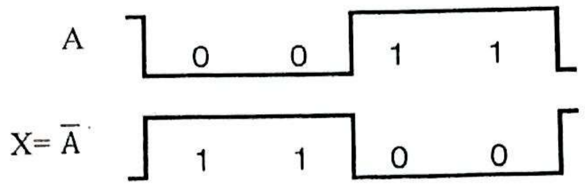
Truth Table for NOT Gate

Input	Output
A	**X= $\overline{A}$**
0	1
1	0

The algebraic expression of the inverter.

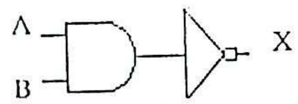
$$X = \overline{A}$$

Figure below shows timing diagram for NOT gate with A = $(1100)_2$, B = $(1010)_2$

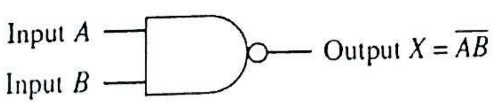


1.4.NAND Gate

This gate can be considered as a NOT-AND gate which is equal to an AND gate followed by a NOT gate. The outputs of all NAND gates are high if any of the inputs are low.



The symbol is NAND gate with a small circle on the output. The small circle represents inversion.

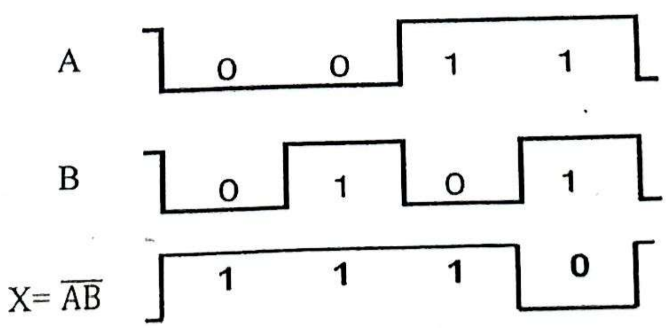


Symbol

Truth table for a Two-Input NAND Gate

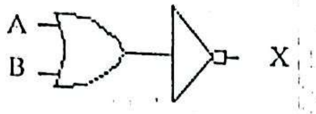
Input		**Output**
A	**B**	**$X = \overline{AB}$**
0	0	1
0	1	1
1	0	1
1	1	0

Figure below shows timing diagram for NAND gate with A = $(1100)_2$, B = $(1010)_2$

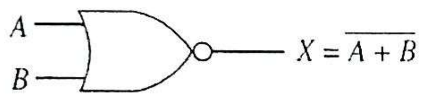


1.5.NOR Gate

This is a NOT-OR gate which is equal to an OR gate followed by a NOT gate. The outputs of all NOR gates are low if any of the inputs are high.



The symbol is NOR gate with a small circle on the output. The small circle represents inversion.

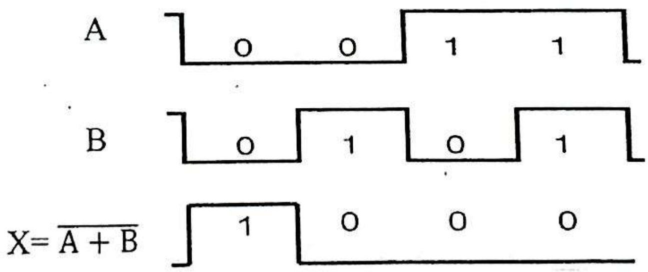


Symbol

Truth table for a Two-Input NOR Gate

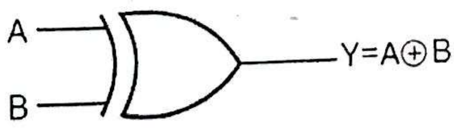
Input		**Output**
A	**B**	**$X = \overline{A+B}$**
0	0	1
0	1	0
1	0	0
1	1	0

Figure below shows timing diagram for NOR gate with A = $(1100)_2$, B = $(1010)_2$



1.6.XOR Gate

The '**Exclusive-OR**' gate is a circuit which will give a high output if **either, but not both**, of its two inputs are high. An encircled plus sign (⊕) is used to show the Exclusive -OR operation.

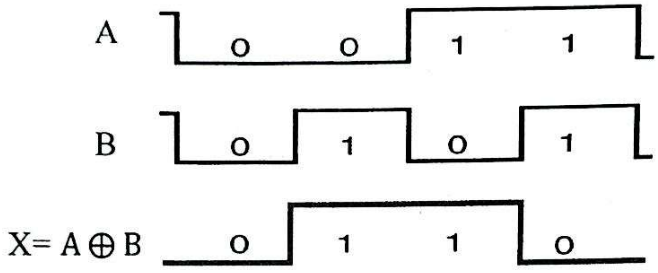


Symbol

Truth table for a Two-Input XOR Gate

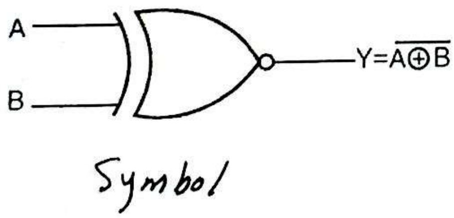
Input		**Output**
A	**B**	**$X = A \oplus B$**
0	0	0
0	1	1
1	0	1
1	1	0

Figure below shows timing diagram for XOR gate with A = $(1100)_2$, B = $(1010)_2$



1.7.XNOR Gate

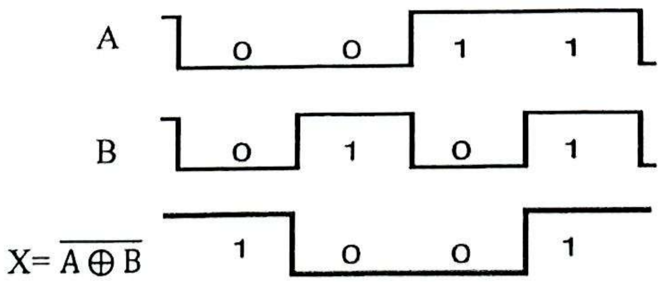
The **'Exclusive-NOR'** gate circuit does the opposite to the EOR gate. It will give a low output if **either, but not both**, of its two inputs are high. The symbol is an EXOR gate with a small circle on the output. The small circle represents inversion.



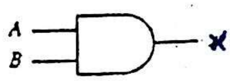
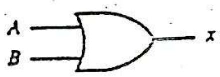
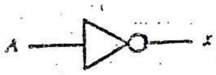
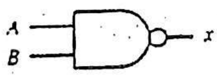
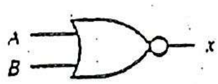
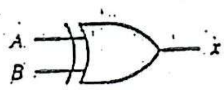
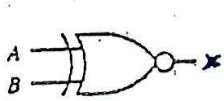
Truth Table for a Two-Input XNOR Gate

Input		Output
A	**B**	**X= $\overline{A \oplus B}$**
0	0	1
0	1	0
1	0	0
1	1	1

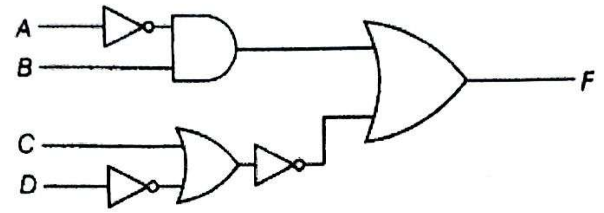
Figure below shows timing diagram for XNOR gate with A = $(1100)_2$, B = $(1010)_2$



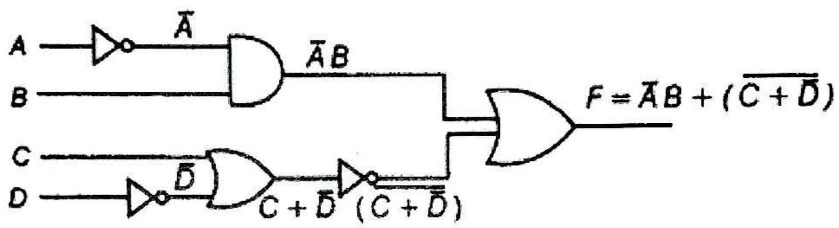
Logic Gats:

Name	Graphic symbol	Algebraic function	Truth table
AND		$x = A \cdot B$ or $x = AB$	A B | x 0 0 | 0 0 1 | 0 1 0 | 0 1 1 | 1
OR		$x = A + B$	A B | x 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 1
NOT (Inverter)		$x = \overline{A}'$	A | x 0 | 1 1 | 0
NAND		$x = \overline{(AB)}'$	A B | x 0 0 | 1 0 1 | 1 1 0 | 1 1 1 | 0
NOR		$x = \overline{(A + B)}'$	A B | x 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 0
Exclusive-OR (X-OR)		$x = A \oplus B$ or $x = \overline{A}B + A\overline{B}$	A B | x 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 0
Exclusive-NOR (X-NOR)		$x = \overline{(A \oplus B)}'$ or $x = \overline{A}\,\overline{B} + AB$	A B | x 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 1

Example: Write the Boolean expression of the logic circuit shown below?

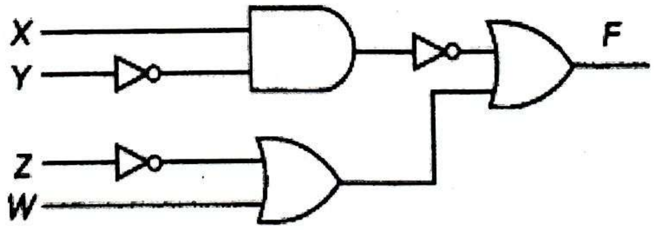


Answer:



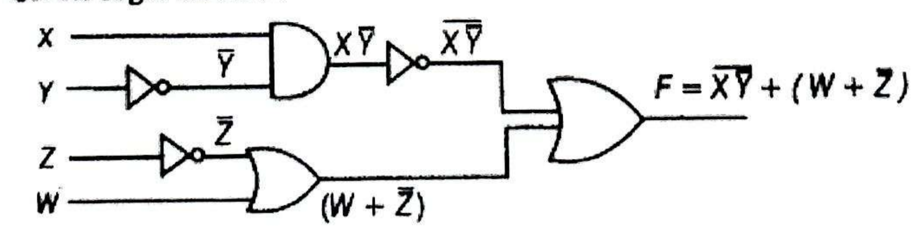
$$F = \overline{A}B + \overline{(C + \overline{D})}$$

Example: Write the Boolean expression of the logic circuit shown below?



Answer:

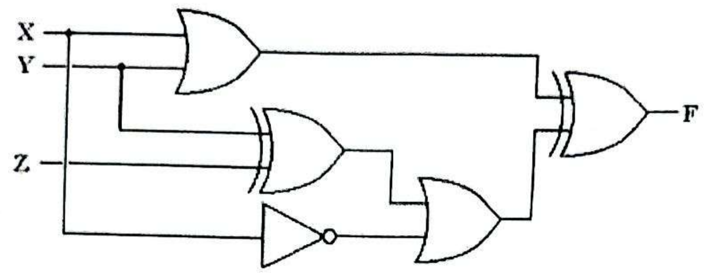
Given logic circuit :



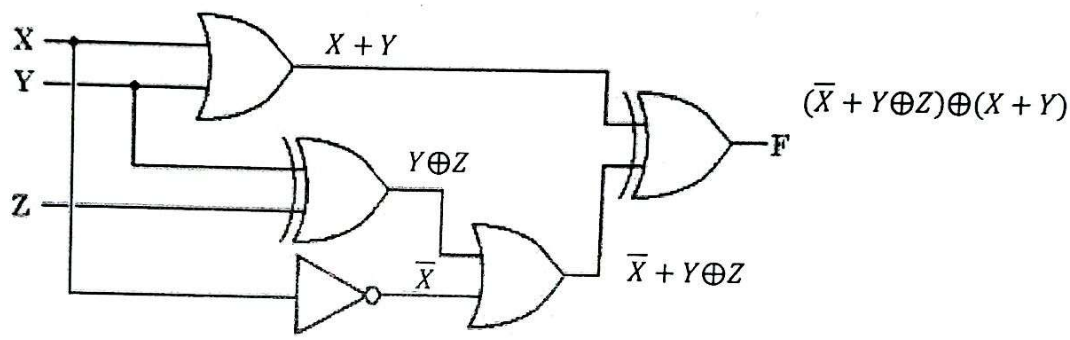
Hence, boolean expression will be $F = \overline{X\overline{Y}} + (W + \overline{Z})$

$$F = \overline{X\overline{Y}} + (W + \overline{Z})$$

Example: Write the Boolean expression of the logic circuit shown below?



Answer:



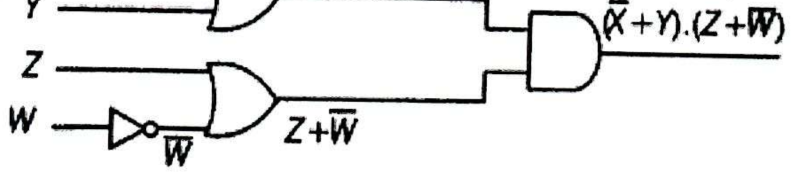
$F = (\overline{X} + Y \oplus Z) \oplus (X + Y)$

Example: Draw the logic circuit for the following Boolean expression:

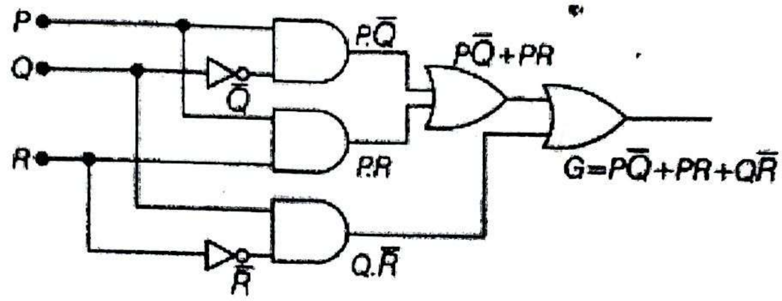
a. $(\overline{X} + Y) . (Z + \overline{W})$
b. $P\overline{Q} + PR + Q\overline{R}$
c. $A\overline{B} + (C + \overline{B})\,\overline{A}$

Answer:

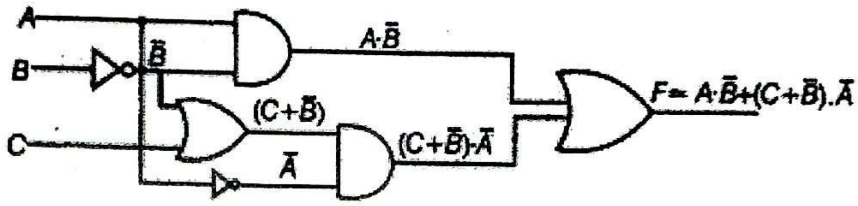
a. The logic circuit for expression $(\overline{X} + Y) . (Z + \overline{W})$ is shown below:



b. The logic circuit for expression $P\overline{Q} + PR + Q\overline{R}$ is shown below:



c. The logic circuit for expression $A\overline{B} + (C + \overline{B})\,\overline{A}$ is shown below:



2. Simplification of Logic Expressions

Boolean algebra finds its most practical use in the simplification of logic circuits. If we translate a logic circuit's function into symbolic (Boolean) form, and apply certain algebraic rules to the resulting equation to reduce the number of terms and/or arithmetic operations, the simplified equation may be translated back into circuit form for a logic circuit performing the same function with fewer components. If equivalent function may be achieved with fewer components, the result will be increased reliability and decreased cost of manufacture.

Table below show some rules used in logic simplification

Logic Simplification Rules

1 →	$A.0 = 0$	10 →	$A.B = B.A$
2 →	$A.1 = A$	11 →	$A + B = B + A$
3 →	$A.A = A$	12 →	$A(B + C) = AB + AC$
4 →	$A.\overline{A} = 0$	13 →	$A + \overline{A}B = A + B$
5 →	$A + 0 = A$	14 →	$\overline{A} + AB = \overline{A} + B$
6 →	$A + 1 = 1$	15 →	A+AB=A
7 →	$A + A = A$		
8 →	$A + \overline{A} = 1$	16 →	(A+B)(A+C)=A+BC
9 →	$\overline{\overline{A}} = A$		

Demorgan's theorem

17 →	$\overline{AB} = \overline{A} + \overline{B}$
18 →	$\overline{A + B} = \overline{A}.\overline{B}$

Prove

13. $A + \bar{A}B = A + B$

$= A \cdot 1 + \bar{A}B = A(1+B) + \bar{A}B$ rule 6

$= A + AB + \bar{A}B = A + B(A + \bar{A})$ rule 8.

$= A + B$

14. $\bar{A} + AB = \bar{A} + B$

$= \bar{A} \cdot 1 + AB = \bar{A}(1+B) + AB$

$= \bar{A} + \bar{A}B + AB = \bar{A} + B(A + \bar{A})$

$= \bar{A} + B$

16. $(A+B)(A+C) = A + BC$

$= (A+B)(A+C) = A \cdot A + AC + AB + BC$

$= A + AC + AB + BC$ rule 3

$= A(1+C) + AB + BC$ rule 6

$= A + AB + BC$

$= A(1+B) + BC$ rule 6

$= A + BC$

Example: Simplify the Boolean expression $X = (A + B)BC + A$ by using simplification rules?

Answer:

$$X = (A + B)BC + A$$

$$X = ABC + BBC + A$$

According to rule 3 we have $B.B = B$ then

$$X = ABC + BC + A$$

$$X = BC(A + 1) + A$$

According to rule 6 we have $A + 1 = 1$ then

$$X = BC + A$$

Example: Simplify the Boolean expression $X = (A + B)\overline{B} + \overline{B} + BC$ by using simplification rules?

Answer:

$$X = (A + B)\overline{B} + \overline{B} + BC$$

$$X = A\overline{B} + B\overline{B} + \overline{B} + BC$$

According to rule 6 we have $B\overline{B} = 0$ then

$$X = A\overline{B} + \overline{B} + BC$$

$$X = (A + 1)\overline{B} + BC$$

According to rule 6 we have $A + 1 = 1$ then

$$X = \overline{B} + BC$$

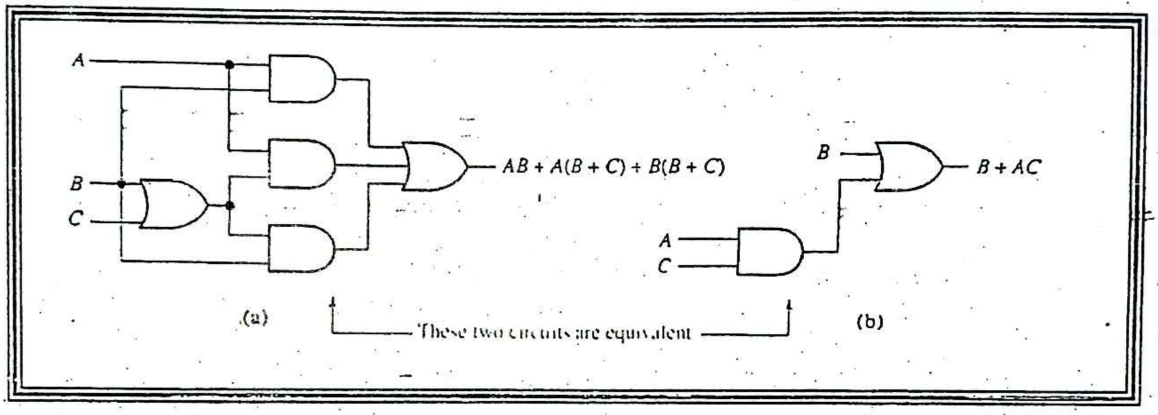
According to rule 14 we have $\overline{B} + BC = \overline{B} + C$ then

$$X = \overline{B} + C$$

Example 2:

AB+ A (B+C)+ B(B+C)

1- AB+AB+AC+BB+BC
2- AB+AB+AC+B+BC
3- AB+AC+B+BC
4- AB+AC+B
5- B+AC

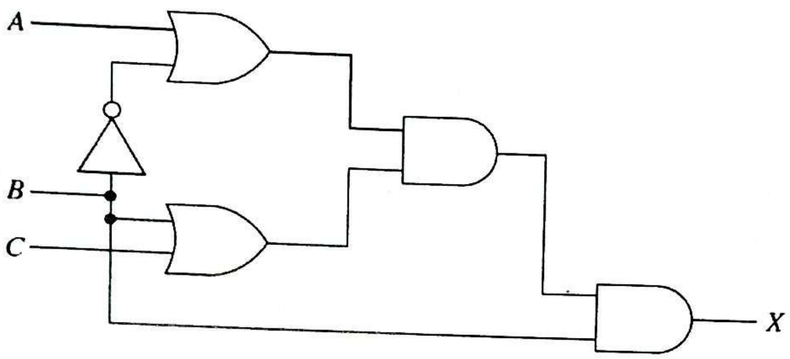


Example: For the logic circuit shown below:

a. Write the Boolean expression of X

b. Simplify the Boolean expression of the logic circuit.

c. Draw the logic circuit of the simplified expression.



Answer:

a. The Boolean expression of X is

$$X = \left((A+\overline{B})(B+C)\right)B$$

b.

$$X = (AB + AC + \overline{B}B + \overline{B}C)B$$

$$X = (AB + AC + \overline{B}C)B$$

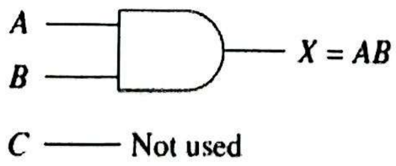
$$X = ABB + ACB + \overline{B}CB$$

$$X = AB + ACB$$

$$X = AB(1+C)$$

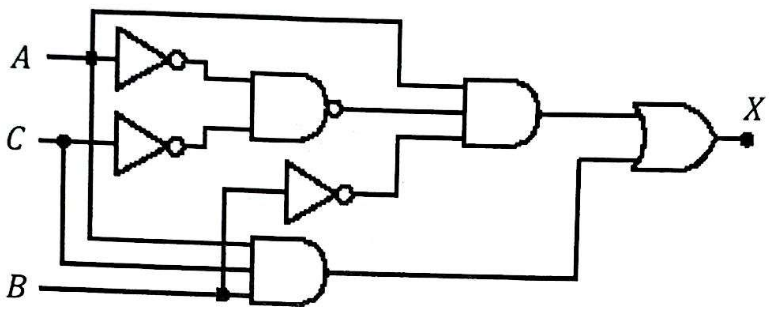
$$X = AB$$

c. The simplified expression



Example: For the logic circuit shown below:

a. Write the Boolean expression of X
b. Simplify the Boolean expression of the logic circuit.
c. Draw the logic circuit of the simplified expression.



Answer:

a. The Boolean expression of X is

$$X = ABC + A\overline{B}(\overline{\overline{A}\,\overline{C}})$$

b.

$$X = ABC + A\overline{B}(\overline{\overline{A}} + \overline{\overline{C}})$$

$$X = ABC + A\overline{B}A + A\overline{B}C$$

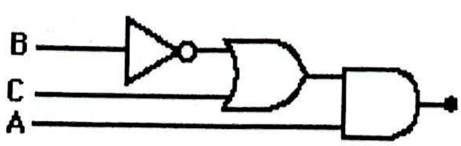
$$X = ABC + A\overline{B} + A\overline{B}C$$

$$X = AC(B + \overline{B}) + A\overline{B}$$

$$X = AC + A\overline{B}$$

$$X = A(C + \overline{B})$$

c. The simplified expression



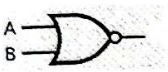
$X = A(C + \overline{B})$

3.De Morgan's Theorem:-

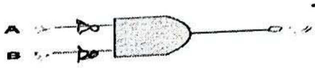
De morgans theorem represent two of the most powerful laws in Boolean algebra.

Law 1: $\overline{A+B}=\bar{A}.\bar{B}$

This states that the complement of sum of variables is equal to the product of their invidual complements.



$\overline{A+B}$ $\cong$



$\bar{A}.\bar{B}$

Truth Table:-

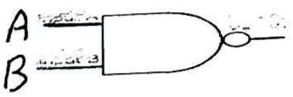
A	B	A+B	$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

A	B	$\bar{A}$	$\bar{B}$	$\bar{A}.\bar{B}$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	0

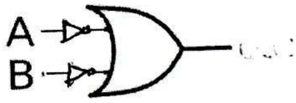
This shows that NOR gate is equivalent to bubbled AND gate

.Law2: $\overline{AB} = \bar{A}+\bar{B}$

This law states that the complement of the product of variables is equal to the sum of their imdividual complements.



$\overline{AB}$ $\cong$



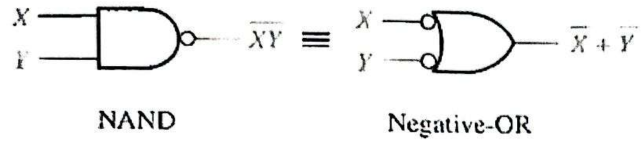
$\bar{A}+\bar{B}$

Truth Table:-

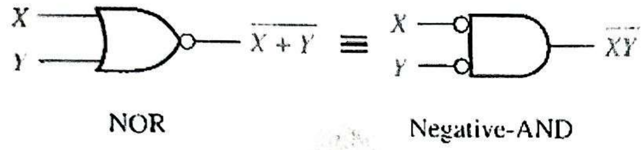
A	B	$\overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0

A	B	$\bar{A}$	$\bar{B}$	$\bar{A}+\bar{B}$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	1
1	1	0	0	0

This shows that NAND gate is equivalent to bubbled OR gate.



Inputs		Output	
X	Y	$\overline{XY}$	$\overline{X}+\overline{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0



Inputs		Output	
X	Y	$\overline{X+Y}$	$\overline{X}\,\overline{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Fig.(4-15) Gate equivalencies and the corresponding truth tables that illustrate DeMorgan's theorems.

As stated, DeMorgan's theorems also apply to expressions in which there are more than two variables. The following examples illustrate the application of DeMorgan's theorems to 3-variable and 4-variable expressions.

Example

Apply DeMorgan's theorems to the expressions $\overline{XYZ}$ and $\overline{X + Y + z}$.

$\overline{XYZ} = \overline{X} + \overline{Y} + \overline{Z}$

$\overline{X + y + Z} = \overline{X}\,\overline{Y}\,\overline{Z}$

Example

Apply DeMorgan's theorems to the expressions $\overline{WXYZ}$ and $\overline{W + X + y + z}$.

$\overline{WXYZ} = \overline{W} + \overline{X} + \overline{y} + \overline{Z}$

$\overline{W + X + y + Z} = \overline{W}\,\overline{X}\,\overline{Y}\,\overline{Z}$

Applying DeMorgan's Theorems

The following procedure illustrates the application of DeMorgan's theorems and Boolean algebra to the specific expression

$$\overline{\overline{(A + B\overline{C})} + D(\overline{E + \overline{F}})}$$

Step 1. Identify the terms to which you can apply DeMorgan's theorems, and think of each term as a single variable. Let $\overline{A + B\overline{C}} = X$ and $D(\overline{E + \overline{F}}) = Y$.

Step 2. Since $\overline{X + Y} = \overline{X}\,\overline{Y}$,

$$\overline{\overline{A + B\overline{C}} + D(\overline{E + \overline{F}})} = (\overline{\overline{A + B\overline{C}}})\,(\overline{D(\overline{E + \overline{F}})})$$

Step 3. Use rule 9 ($\overline{\overline{A}} = A$) to cancel the double bars over the left term (this is not part of DeMorgan's theorem).

$$(\overline{\overline{A + B\overline{C}}})\,(\overline{D(\overline{E + \overline{F}})}) = (A + B\overline{C})(\overline{D(\overline{E + \overline{F}})})$$

Step 4. Applying DeMorgan's theorem to the second term,

$$(A + B\overline{C})(\overline{D(\overline{E + \overline{F}})}) = (A + B\overline{C})(\overline{D} + (\overline{\overline{E + \overline{F}}}))$$

Step 5. Use rule 9 ($\overline{\overline{A}} = A$) to cancel the double bars over the $E + \overline{F}$ part of the term.

$$(A + B\overline{C})(\overline{D} + \overline{\overline{E + \overline{F}}}) = (A + B\overline{C})(\overline{D} + E + \overline{F})$$

Example

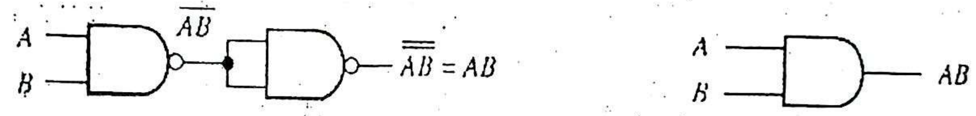
Apply DeMorgan's theorems to each of the following expressions:

(a) $\overline{(A + B + C)D}$ (b) $\overline{ABC + DEF}$ (c) $\overline{A\overline{B} + C\overline{D} + EF}$

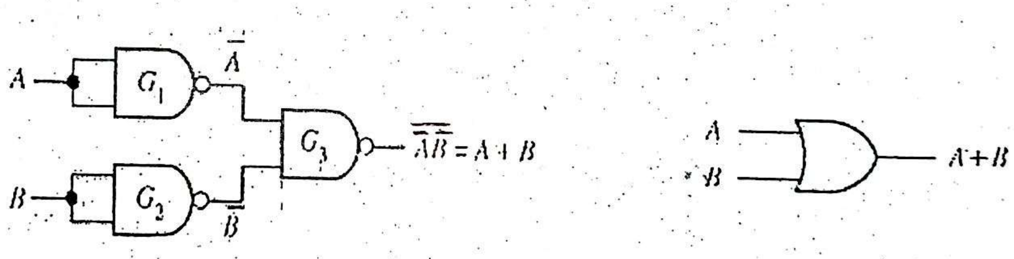
Universal Building Blocks

1-The NAND Gate as a Universal Logic Element:

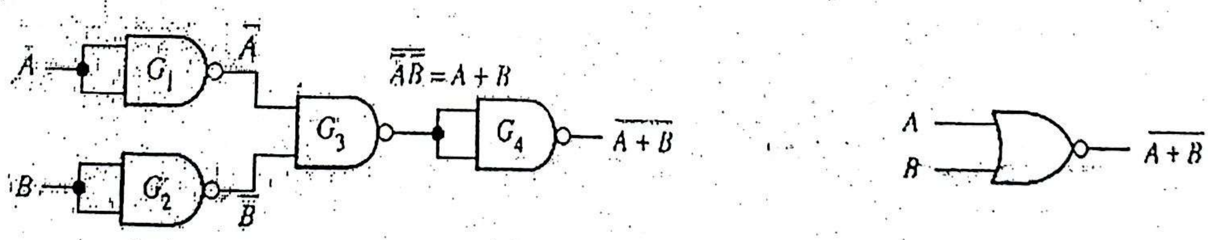
(a) A NAND gate used as an inverter



(b) Two NAND gates used as an AND gate



(c) Three NAND gates used as an OR gate

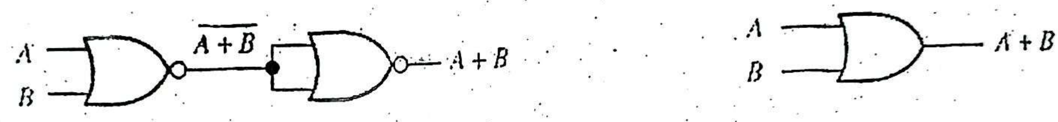


(d) Four NAND gates used as a NOR gate

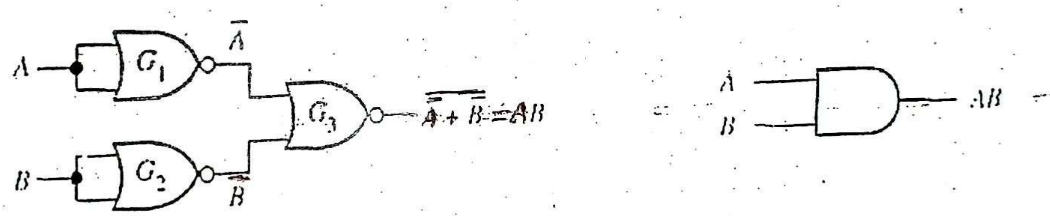
Figure (9) NAND Gates

2-The NOR Gate as a Universal Logic Element:

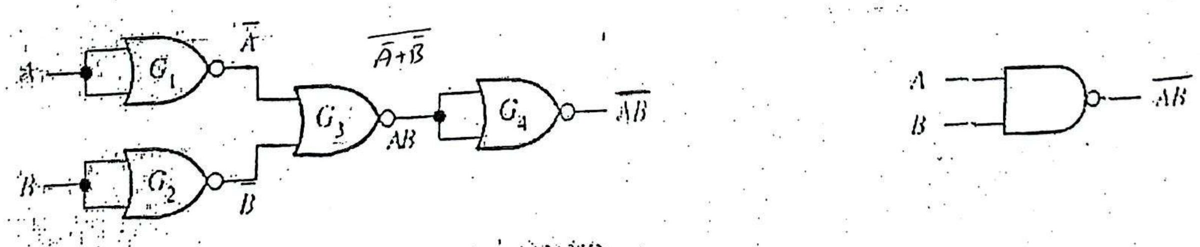
(a) A NOR gate used as an inverter



(b) Two NOR gates used as an OR gate



(c) Three NOR gates used as an AND gate



(d) Four NOR gates used as a NAND gate

Figure (10) NOR Gates

3.1.Min Term & Max term

Min term means the term that is true for a minimum number of combination of inputs. That is true for only one combination of inputs. Since AND gate also gives True only when all of its inputs are true so we can say min terms are AND of input combinations like in the table given below.

Maxterm means the term or expression that is true for a maximum number of input combinations or that is false for only one combination of inputs.

For Min term we have

- $A \rightarrow 1$, $\overline{A} \rightarrow 0$
- Min terms of each output is the product of all it's input

For Max term we have

- $A \rightarrow 0$, $\overline{A} \rightarrow 1$
- Max terms of each output is the sum of all it's input

Table below shows example of Minterms and Maxterms

A	B	C	Minterm	Maxterm	Output
0	0	0	$\overline{A}.\overline{B}.\overline{C}$	$A+B+C$	0
0	0	1	$\overline{A}.\overline{B}.C$	$A+B+\overline{C}$	1
0	1	0	$\overline{A}.B.\overline{C}$	$A+\overline{B}+C$	0
0	1	1	$\overline{A}.B.C$	$A+\overline{B}+\overline{C}$	1
1	0	0	$A.\overline{B}.\overline{C}$	$\overline{A}+B+C$	0
1	0	1	$A.\overline{B}.C$	$\overline{A}+B+\overline{C}$	1
1	1	0	$A.B.\overline{C}$	$\overline{A}+\overline{B}+C$	0
1	1	1	$A.B.C$	$\overline{A}+\overline{B}+\overline{C}$	0

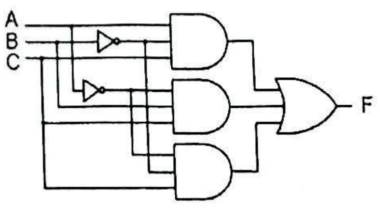
To obtain the Boolean expression of the Output we can use either SOP (Sum-of-Product) or POS (Product-of-Sum) as follows:

- SOP method:

The output is sum of all Minterms for outputs that generate 1 as shown below.

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

$$Y = A\cdot\overline{B}\cdot C + \overline{A}\cdot B\cdot C + \overline{A}\cdot\overline{B}\cdot C$$



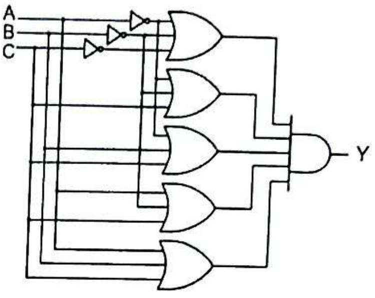
We can write the output as F(A, B, C) = Σ(1, 3, 5)

- POS method:

The output is sum of all Maxterms for outputs that generate 0 as shown below.

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

$$Y = (\overline{A}+\overline{B}+\overline{C})\cdot(\overline{A}+\overline{B}+C)\cdot(\overline{A}+B+C)\cdot(A+\overline{B}+C)\cdot(A+B+C)$$



We can write the output as F(A, B, C) = Π(0, 2, 4, 6, 7)

Example: Find a logic expression for:

a. F(A, B, C) = Σ(1, 4, 7)
b. F(A, B, C) = Π(0, 2, 4, 5)

Answer:

a. First we obtain the truth table of F(A, B, C) = Σ(1, 4, 7)

Decimal	A	B	C	Minterm	Output
0	0	0	0		0
1	0	0	1	$\overline{A}.\overline{B}.C$	1
2	0	1	0		0
3	0	1	1		0
4	1	0	0	$A.\overline{B}.\overline{C}$	1
5	1	0	1		0
6	1	1	0		0
7	1	1	1	$A.B.C$	1

then

$$\text{Output} = \overline{A}.\overline{B}.C + A.\overline{B}.\overline{C} + A.B.C$$

b. The truth table of $F(A, B, C) = \Pi(0, 2, 4, 5)$ is

A	B	C	Maxterm	Output
0	0	0	$A + B + C$	0
0	0	1		1
0	1	0	$A + \overline{B} + C$	0
0	1	1		1
1	0	0	$\overline{A} + B + C$	0
1	0	1	$\overline{A} + B + \overline{C}$	0
1	1	0		1
1	1	1		1

then

$$\text{Output} = (A + B + C) . (A + \overline{B} + C) . (\overline{A} + B + C) . (\overline{A} + B + \overline{C})$$

Example: Find a logic expression for:

a. $F(A, B, C, D) = \Sigma(1, 4, 5, 8)$ in S.O.P & P.O.S

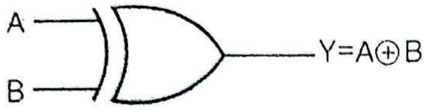
Answer:

$$\text{Output} = \overline{A} . \overline{B} . \overline{C} . D + \overline{A} . B . \overline{C} . \overline{D} + \overline{A} . B . \overline{C} . D + A . \overline{B} . \overline{C} . \overline{D}$$ S.O.P

$$\text{Output} = (A + B + C + D) . (A + B + \overline{C} + D) . (A + B + \overline{C} + \overline{D}) . (\overline{A} + B + C + \overline{D})$$ P.O.S

XOR Gate

The '**Exclusive-OR**' gate is a circuit which will give a high output if **either, but not both**, of its two inputs are high. An encircled plus sign (⊕) is used to show the Exclusive -OR operation. The XOR gate has only two inputs.



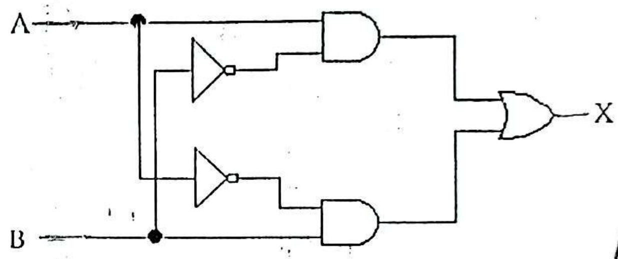
Symbol

Truth table for a Two-Input XOR Gate

Input		Output
A	B	X= A ⊕ B
0	0	0
0	1	1
1	0	1
1	1	0

From the truth table :

$$X = A\overline{B} + \overline{A}B = A \oplus B$$



XNOR Gate

The XNOR gate has only two inputs as in the case of XOR gate. The XNOR gate is the complement of the XOR gate for the same inputs.

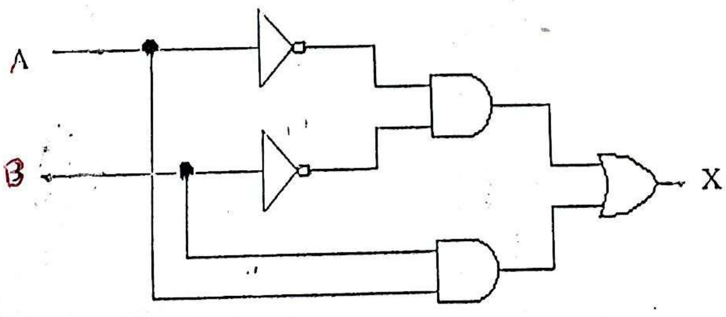
Symbol

The Truth Table of XNOR gate.

Input		Output
A	B	X
0	0	1
0	1	0
1	0	0
1	1	1

From the truth table :

$$X = \overline{A}\,\overline{B} + AB = \overline{A \oplus B}$$



4. Exercises

1. Simplify the following logic expression using simplification rules:

   a. $X = A\bar{B}C + ABC + (C + D)(\bar{D} + E)$.

   b. $X = \overline{(A\bar{B} + \bar{A}B)}(A + B)$

   c. $X = (\bar{A} + B)(A + B + C)\bar{C}$

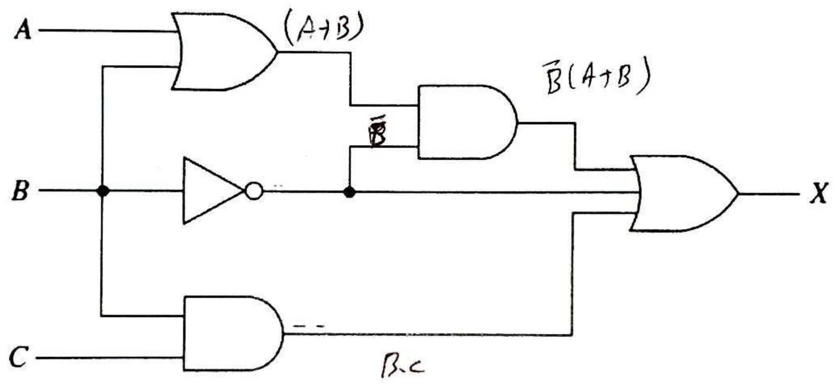
   d. $X = ABC + A\bar{B}C + AB\bar{C}$

2. For the logic circuits shown below :

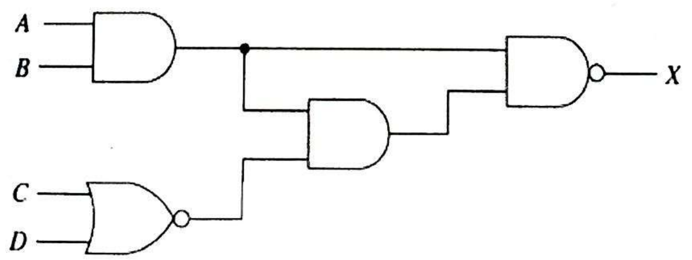
   a. Write the Boolean expression of X.

   b. Simplify the Boolean expression of the logic circuit.

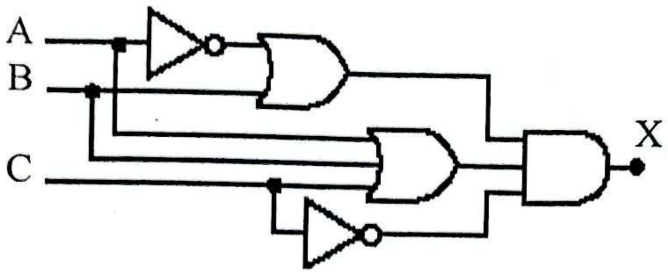
   c. Draw the logic circuit of the simplified expression.



(i)



(ii)



(iii)

3. Design the logic circuit from the truth table shown below by using:
   a. SOP.
   b. POS.

A	B	C	Output
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

4. Design logic circuits with SOP and POS for each of the following:
   a. A logic circuit that gives 1 if the input is odd number (use 3 digit as input).
   b. A logic circuit that gives 1 if the input >= 4 (use 3 digit as input).
   c. A logic circuit that gives 1 if the input equals 1,5,8 (use 4 digit as input).