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قسم هندسة تقنيات الحاسوب

محاضرات القياسات

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مدرس المادة

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System of Units

The principle aspects of the scientific method are accurate measurement, selective analysis, and mathematical formulation. Note that the first and most important is accurate measurements.

***Measurement*:** *is the process by which one can convert physical parameters to meaningful number.*

Instrument: *may be defined as a device for determining the value or magnitude of a quantity or variable.*

The standard measure of each kind of physical quantity is the unit; the number of times the unit occurs in any given amount of the same quantity is the number of measure. With out the unit, the number of measure has no physical meaning.

Fundamental and Derived Units

To measure an unknown we must have acceptable unit standard for the property that is to be assessed. Since there are virtually hundreds of different quantities that man is called upon to measure, it would seem that hundreds of different standard units would be required. Fortunately, this is not the case. By choosing a small number of basic quantities as standards, we can define all the other in terms of these few.

The basic units are called ***fundamentals***, while all the others which can be expressed in terms of fundamental units are called ***derived*** units, and formed by multiplying or dividing fundamental units. The ***primary fundamental*** units which most commonly used are ***length, mass, and time,*** while measurement of certain physical quantities in ***thermal, electrical, and illumination*** disciplines are also represented by fundamental units. These units are used only when these particular classes are involved, and they may therefore be defined as ***auxiliary fundamental*** units. Every derived unit originates from some physical law defining that unit. For example, the voltage [volt]:

$$volt = \frac{workdone}{charge} = \frac{Joule}{coulomb} = \frac{J}{C} = \frac{Force \times distance}{current \times time} = \frac{Newton \times meter}{Amper \times second} \Rightarrow$$

$$volt = \frac{mass \times acceleration \times meter}{current \times time} = \frac{mass \times \frac{velocity}{time} \times meter}{current \times time} = \frac{mass \times \frac{distance}{time^2} \times meter}{current \times time}$$

$$volt = \frac{mass \times \frac{meter^2}{time^2}}{current \times time} = \frac{mass \times meter^2}{current \times time^3} = \frac{Kg.m^2}{A.\sec^3} = \left[Kg.m^2.A^{-1}.\sec^{-3}\right] \text{ basic S.I units}$$

A derived unit is recognized by its ***dimensions***, which can be defined as the complete algebraic formula for the derived unit. The dimensional symbols for the fundamental units of length, mass, and time are **L, M,** and **T**, respectively. So the dimensional symbol for the derived unit of voltage is $V = \frac{M.L^2}{I.T^3} = \left[M.L^2.I^{-1}.T^{-3}\right]$

Table (1) shows the six basic ***S.I*** quantity and units of measurement, with their unit symbol:

Table (1):

Quantity	*Unit*	*Symbol*
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electrical current	Ampere	A
Thermodynamic temperature	Kelvin	K
Luminous intensity	Candela	cd

Table(2) : shows the development of system of units since 1790 to our days

Quantity	*Dimensional symbol*	*British*	*CGS*	*CGSe*		*CGSm*		*MKS*	*MKSA S.I units*
length	L	ft	cm	cm	+Є$_o$=1 for free space	cm	+μ$_o$=1 for free space	m	m
mass	M	lb	g	g		g		kg	kg
time	T	sec	sec	sec		sec		sec	sec
current	I								Amp

The CGS electrostatic system ***(CGSe)*** used coulomb's law for the force between two electric charges. $F = k\dfrac{Q_1 Q_2}{r^2}$, and assume k= 1/Є$_o$ to find the basic S.I units for electric charge Q which equal ($cm^{3/2} g^{1/2} s^{-1}$), then from electric charge all electrical units (I, V, E, C...) are determined by their defining equations. The same things is depended in CGS electromagnetic system ***(CGSm)***, but at this time coulomb's law determine the force between two magnetic poles with proportionality factor k=1/ μ$_o$ to derived electromagnetic unit of polestrength (m) which equal ($cm^{3/2} g^{1/2} s^{-1}$) then determine all other magnetic units (B, H, Φ, ...). ***Rationalised system*** of units used (MKS) system and assignicant the value of $\mu_0=4\pi x10^{-7}$ *H/m*, and $Є_o=8.85x10^{-14}$ *F/m.*

Multiples and Submultiples of units

The units in actual use are divided into submultiples for the purpose of measuring quantities smaller than the unit itself. Furthermore, multiples of units are designated and named so that measurement of quantities much larger than the unit is facilitated. Table(3) lists the decimal multiples and submultiples of units.

Table(3):

Name	*Symbol*	*Equivalent*
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	K	10^{3}
milli	m	10^{-3}
micro	µ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}

Basic Definitions:

1. ***Speed, Velocity***: the rate of change of distance with respect to time

$$\upsilon = \frac{\partial x}{\partial t} \quad , \qquad \chi = \int_0^t \upsilon \partial t = \upsilon . t \quad , \qquad \upsilon = \frac{\chi}{t}$$

$\upsilon = [LT^{-1}]$ basic dimensions, $\upsilon = [m\sec^{-1}]$ basic S.I units

2. ***Acceleration:*** the rate of change of velocity during the time

$$a = \frac{\partial \upsilon}{\partial t}, \qquad \upsilon = \int_0^t a \partial t = a.t \quad , \qquad a = \frac{\upsilon}{t}$$

$a = [LT^{-2}]$ basic dimensions, $a = [m\sec^{-2}]$ basic S.I units

3. ***Momentum:***

$$p = mass \times velocity = m \times \upsilon$$

$p = [MLT^{-1}]$ basic dimensions , $p = [kgm\sec^{-1}]$ basic S.I units

4. ***Force:*** (Newton), the rate of change of momentum during the time

$F = \frac{\partial p}{\partial t} = \frac{\partial (m\upsilon)}{\partial t}$, $F = [MLT^{-2}]$ basic dimensions, $F = [kgm\sec^{-2}]$ basic S.I units

5. ***Energy:*** (Joule), the distance integral of force

$$E = \int_0^{\chi} F \partial \chi = F \cdot \chi$$

$E = [ML^2T^{-2}]$ basic dimensions, $E = [kgm^2\sec^{-2}] = Joule = J$

6. ***Power:*** (Watt) , the rate of work done

$$P = \frac{\partial E}{\partial t}$$

$P=\left[ML^2T^{-3}\right]$ basic dimensions, $P=\left[kgm^2\sec^{-3}\right]$ S.I units, $P=J.\sec^{-1}$

7. **Potential of a point (voltage):** work done to bring a unit charge from infinity to same point.

$$V=\frac{workdone}{charge}=\frac{Joule}{coulomb}$$

$V=\left[ML^2I^{-1}T^{-3}\right]$ basic dimensions, $V=\left[kgm^2A^{-1}\sec^{-3}\right]$ basic S.I units

8. **Electrical current:** the rate of flow of charge

$$I=\frac{\partial Q}{\partial t},\quad Q=\int_0^t I\partial t,\ Q=I.t$$

$I=\left[Amp\right]$

9. **Resistance (ohm):** the resistance of a load to the current flow when there is voltage difference between its terminals.

$R=\frac{\partial V}{\partial I}$, $R=\left[ML^2I^{-2}T^{-3}\right]$ dimensions, $R=\left[kgm^2A^{-2}\sec^{-3}\right]$ basic S.I units

10. **Capacitance (farad):**

$C=\varepsilon\frac{A}{d}$, or $C=\frac{Q}{V}$, $C=\left[M^{-1}L^{-2}I^2T^4\right]$, $C=\left[kg^{-1}m^{-2}A^2\sec^4\right]$

11. **Electrical field:**

$E=\frac{\partial V}{\partial x}$, $E=\left[MLI^{-1}T^{-3}\right]$, $E=\left[kgmA^{-1}\sec^{-3}\right]$

12. **Permittivity Є:** how much electrical field lines can pass through some medium

$Є=\frac{farad}{m}$, $Є=\left[M^{-1}L^{-3}I^2T^4\right]$, $Є=\left[kg^{-1}m^{-3}A^2\sec^4\right]$

13. **Inductance(henry):**

Induce emf = inductance x rate of change of current

$$e=-L\frac{\partial i}{\partial t},\qquad \int_0^t e\partial t=L\int_0^i \partial i,\qquad L=\frac{et}{I}$$

$Henry=\left[ML^2I^{-2}T^{-2}\right]$, $Henry=\left[kgm^2A^{-2}\sec^{-2}\right]$

14. **Reluctance (S):** the magnetic resistance to magnetic field lines in same material

$S=\frac{l}{\mu.A}$, $S=\left[M^{-1}L^{-2}I^2T^2\right]$, $S=[kg^{-1}m^{-2}A^2\sec^2]$

15. ***Magnetic flux(Φ) weber:***

$$\phi = \frac{mmf}{S} = \frac{N.I}{S}, \ \phi = \left[ML^2 I^{-1} T^{-2}\right], \ \phi = \left[kgm^2 A^{-1} \sec^{-2}\right]$$

16. ***Frequency(hertz):*** number of cycles in one second

$$f = \frac{cycles}{second} = \frac{1}{\sec}, \quad f = \left[T^{-1}\right], \ f = \left[\sec^{-1}\right]$$

17. ***Light speed (c):***

a) Speed of light in free spaces $c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$

b) Speed of light in same medium $\upsilon = \frac{1}{\sqrt{\mu\varepsilon}}$

c) Diffraction factor $N = \frac{c}{\upsilon}$

Notes that constant and numbers have no units (unit less)

Standard of Measurements

A standard of measurement is a physical representation of a unit of measurement. A unit is realized by reference to an arbitrary material standard or to natural phenomena including physical and atomic constants.

Standard of measurement classified by their function and application in the following categories:

1. International standards
2. Primary standards
3. Secondary standards
4. Working standards

The international standards are defined by international agreement. They represent certain units of measurements to the closest possible accuracy that production and measurement technology allow. These standards are maintained at the International Bureau of Weights and Measures in America and not available to the ordinary user of measuring instruments.

The primary (basic) standards are maintained by national standards laboratories in different parts of the world. The National Bureau of standards (NBS) in America, National Physical Laboratory (NPL) in Britain, and Physikalisch Technische in Germany. The primary standards represent the fundamental units and some of the derived mechanical and electrical units. Primary standards are not available for use outside the national laboratories. One of the main functions of primary standards is the verification and calibration of secondary standards.

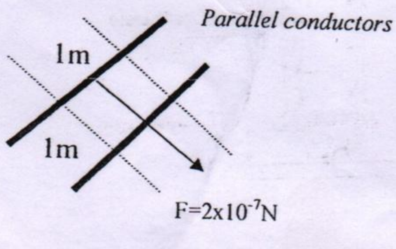
Secondary standards are the basic reference standards used in industrial measurement laboratories. These standards are maintained by the particular involved industry and are generally sent to the national standards laboratories (primary) on a periodic basis for calibration and comparison.

Working standards are the principal tools of a measurement laboratory. They are used to check and calibrate general laboratory instrument for accuracy and performance or to perform comparison measurements in industrial applications. A manufacturer of precision resistances, for example, may use a standard resistor (a working standard) in the quality control department of his plant to check his testing equipment.

Electrical Standards

1. ***The Absolute Ampere***

The international system of units (S.I) defines the ampere (the fundamental unit of electrical current) as the constant current which, if maintained in two straight parallel conductors of infinite length and negligible circular cross section placed (1m) apart in a vacuum, will produce between these conductor a force equal to $2x10^{-7}$ Newton/meter.



The international Ampere was then defined as that current which deposits silver at the rate of (1.11mg/sec) from a standard silver nitrate solution. The international Ampere was superseded by the absolute Ampere which is determined by Reyleigh current balance. The force acting on the moving coil, and measured by balance is given by:

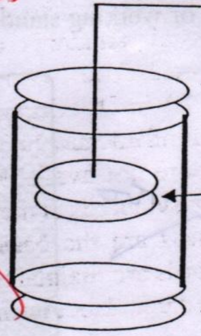
$$F = I^2 \frac{\partial M}{\partial x} \quad \text{Newton}$$

where I: current in Amp in three series coils.

M: mutual inductance of the coils depends on number of turns, dimensions, and relative positions.

∂x: element of the length a long the axis of the three coils.

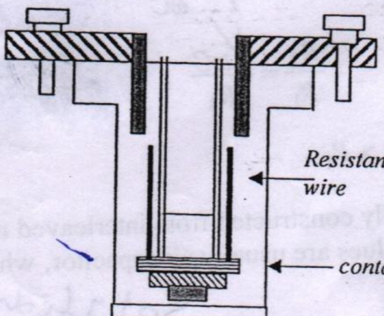
If the moving coil is at a distance of half their radius from each of the fixed coil, the $\partial M/\partial x$ depend on the ratio of: $\frac{\text{radius of fixed coil}}{\text{radius of moving coil}}$



2. Resistance Standards:

The absolute measurement of the ohm is carried out by the international Bureau of weights and Measures and also by the national standards laboratories (NBS) maintains (1Ω standards resistors).

The standard resistor is a coil of wire of some alloy like manganin (alloy of Nickel, manganese, and copper) which has a high electrical resistivity and low temperature coefficient of resistance. The resistance coil is mounted in a double walled sealed container to prevent changes in resistance due to moisture conditions in the atmosphere. Secondary standards and working standards are available from some instrument manufacturers in a wide range of values usually in multiples of 10Ω.



3. Voltage Standards:

For many years the standards voltage was based on an electrochemical cell called the saturated standard cell or standards cell, which also called a ***Saturated Weston Cell.*** The Weston cell has appositive electrode of mercury and a negative electrode of cadmium amalgam. The electrolyte is a solution of cadmium sulphate. These components of are placed in an H-shaped glass container. There are two type of Weston cell:

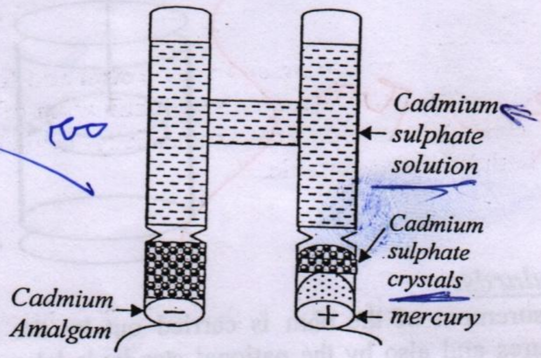
a) Saturated cell which has a voltage variation of approximately (-40 μV) per 1ºC rise.

b) The Unsaturated cell which has a negligible temperature coefficient of voltage at normal room temperature, but saturated at 4ºC.

The voltage of the Weston saturated cell at 20 ºC is (1.01858V) absolute, and the e.m.f at the other temperature is given by the formula:

$$V(t)=V_{20C}-46\times10^{-6}(t-20)-95\times10^{-8}(t-20)^2+1\times10^{-8}(t-20)^3$$

Since saturated cells are temperature sensitive, they are unsuited for general laboratory use as secondary or working standards, while unsaturated cells are used.



4. Capacitance standards:

The unit of capacitance (farad) can be measured with a Maxwell d.c commutated bridge, where the capacitance is computed from the resistive bridge arms and the frequency of d.c commutation. $I=\omega.Q$, where ω is the frequency of commutator

$$C=\frac{Q}{V}\rightarrow Q=C.V\Rightarrow I=\omega.C.V \quad \Rightarrow\frac{V}{I}=\frac{1}{\omega C}=Xc$$

At balance $\frac{R_2}{R_1}=\frac{R_4}{R_3}$, $\frac{R_2}{R_1}=\frac{1/\omega C}{R_3}$, $\frac{R_1}{R_2}$

$$C=\frac{R_1}{\omega R_2 R_3},\quad \text{where } R_2+R_3>R_1$$

Standards capacitors are usually constructed from interleaved metal plates with air as the dielectric material. Smaller values are usually air capacitor, whereas the larger capacitors use solid dielectric materials.

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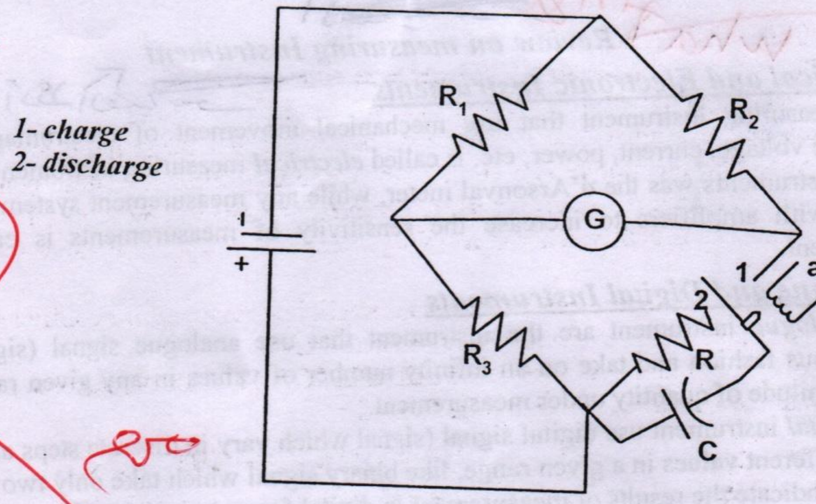


5. Inductance Standards:

The primary inductance standard is derived from the ohm and farad rather from the large geometrically constructed inductors used in the determination of the absolute value of the ohm. Inductance working standards are commercially available in a wide range of practically values, both fixed and variable.

6. IEEE Standards:

A slightly different type of standard is published and maintained by (***Institute of Electrical and Electronics Engineers***).

These standards are not physical items that are available for comparison and checking of secondary standards, but are ***standard procedures, nomenclature, and definitions.***

A large group of the IEEE standards is the standard test methods for testing and evaluating various electronics systems and components such as:

1- Standard method for testing and evaluating attenuators.
2- Standards for specifying test equipment, like IEEE standard addresses the laboratory oscilloscope and specifies the controls, functions, etc.
3- Various standards concerning the safety of wiring for power plants, ships, industrial buildings.
4- The most important standards is (IEEE 488) digital interface for programmable instrumentation for test and other equipment.

Review on measuring Instrument

1- *Electrical and Electronic Instruments*

The measuring instrument that use mechanical movement of electromagnetic meter to measure voltage, current, power, etc. is called ***electrical*** measuring instrument, so the heart of these instruments was the d'Arsonval meter, while any measurement system use d'Arsonval meter with amplifiers to increase the sensitivity of measurements is called *electronic* instrument.

2- *Analogue and Digital Instruments*

An ***analogue*** instrument are the instrument that use analogue signal (signal varying in continuous fashion and take on an infinity number of values in any given range) to display the magnitude of quantity under measurement.

The ***digital*** instrument use digital signal (signal which vary in discrete steps and take up only finite different values in a given range, like binary signal which take only two levels zero and one) to indicate the results of measurement in digital form.

3- *Absolute and Secondary Instruments*

In ***absolute*** instrument the measured value is given in term of instrument constants and the deflection of one part of the instrument e.g. tangent galvanometer, and Rayliegh current balance. In these instruments no calibrated scale is necessary. While in ***secondary*** instruments, the quantity of the measured values is obtain by observing the out put indicate by these instruments.

Classification of Secondary Instruments

a) *Indicating Instruments*

The magnitude of quantity being measured is obtain by deflection the pointer on scale, and the output is indicate either in analogue or digital form like *ammeter, voltmeter, and wattmeter.*

Three forces was acting on the pointer to deflect it in proportional to the quantity being measured, these forces are:

i) *Deflecting Force*

This force gives the pointer the initial force to move it from zero position, its also called operating force.

ii) *Controlling Force*

This force control and limits the deflection of the pointer on scale which must be proportional to the measured value, and also ensure that the deflection is always the same for the same values.

iii) *Damping Force*

This force is necessary in order to bring the movement system (pointer) to rise quickly to the measured value, and then stop without any oscillation.

b) *Recording Instruments*

An instrument which makes a written record in any recorded medium to the quantity being measured in order to save information and use it in anther time or anther place. Recording instrument may record transient signal, or phenomena which can not obtain readily. This instruments like *recording devices, X-Y plotter, and oscilloscope.*

c) *Controlling Instruments*

These instruments give an information or instruction (orders) to control on original measured quantity or control on other devices, like *a computer*.

Factors Effecting Instrument selection

1- *Accuracy*

Its represent how ***closeness*** with which an ***instrument reading*** approaches the ***true value*** of the variable being measured.

The deviation of the measured value from the true value is the indication of how accurately reading has been made.

2- *Precision*

It's specified the ***repeatability*** of a set of reading each made independently with the same instrument.

An estimate of precision is determined by the deviation of different reading from the mean (average) value.

Example:

To detect the deference between accuracy and precision of measurement for some voltage, we see the following cases:

i) V=6Volt (true or theoretical value) V=5.8Volt (measured or practical value)
This instrument is accurate

ii) V=6Volt (true or theoretical value) V=4.8Volt (measured or practical value)
This instrument is not accurate

iii) V=6Volt (true or theoretical value) V=5.8Volt (measured or practical value)
When we try to check the reading, we measured it again and again, and get the following results: second measure for the same reading equal V=5.8Volt, third measured V=5.8Volt, forth measured V=5.8Volt and so on.
This instrument is accurate and precise

iv) V=6Volt (true or theoretical value) V=4.8Volt (measured or practical value)
We try to check the reading, we measured it again and again, and get the following results: second measure for the same reading equal V=5Volt, third measured V=4.6Volt, forth measured V=5.2Volt and so on. This instrument is not accurate and not precise.

3- *Range*

It is defined as that region enclosed by the limits within which a particular quantity is measured.

4- *Span*

It is algebraic difference of the upper and lower limits of the range.

Example:

The span of (0 to10) voltmeter is Span= 10-0=10 state

But the span for (-10 to +10) voltmeter is Span= 10-(-10) =20 state

5- *Loading effect*

It's the change of circuit parameter, characteristic, or behaves due to instrument operation with out maintains.

6- *Sensitivity*

It's represent the ratio of output signal to a change in input, or its represent the response output of the instrument to a change of its input.

7- *Resolution*

The smallest change in input that the instrument can response to it, or the ratio of output to smallest change in input.

8- *Error*

The deviation of the measured value from the true value.

Types of Errors

No measurement can be made with perfect accuracy, but its important to find out what the accuracy actually is , and who different errors have enter to the measurement, so study of errors is a first steps in finding ways to reduce them.

Errors may come from different sources and are usually classified under two main heading:

1- *Systematic Errors*

These types of errors have known reasons, and we can avoided, reduce or eliminated, and estimated it. These errors are subdivided into:

a) *Gross (Human) Errors*

- **i)** Misreading of instruments and observation errors.
- **ii)** Improper choice of instrument, or the range of instrument.
- **iii)** Incorrect adjustment or forgetting to zero.
- **iv)** Erroneous calculations, computation mistakes, and estimation errors.
- **v)** Neglect of loading effects.
- **vi)** Proper position for measuring human.

b) *Instrumentation (Equipment) Errors*

- **i)** Damaged equipment such as defective due to loading effect or worn parts.
- **ii)** Calibration errors.
- **iii)** Bearing fraction.
- **iv)** Component nonlinearities.
- **v)** Loss during transmission.
- **vi)** Proper position of equipment (vertical or horizontal).
- **vii)** Static charge error.

c) *Environmental Errors*

- **i)** Change in temperature, pressure.
- **ii)** Humidity.
- **iii)** Stray electric and magnetic fields.
- **iv)** Mechanical vibration.
- **v)** Weather variations (day, night, and four seasons).

d) *Measuring Errors*

Measuring human does not have enough efficiency and experience to expect the true measurement values and the reasons of errors.

2- *Random Errors*

Those due to causes that can not be directly established because of unknown events that causes small variation in measurement, quite random and unexplained. We can reduce this type of errors after treatment the systematic errors by taking many reading for the measuring value and apply statistical analysis to determine the best true estimate of measurement readings.

Example (1):

(Systematic, Human errors, the proper range of measurement)

A 0 to 150V voltmeter has accuracy of 1% of full scale reading. The theoretical (true) expected value we want to measure it is 83V. Determine the practical (measured) value and the percentage of error.

Sol.:

Tolerance= accuracy x V_{FSD}
Tolerance= 1% x 150 = 0.01 x 150 = 1.5V
Measured value= true ± tolerance
Measured value= 83 ± 1.5
Measured value= 84.5V or 81.5V

The percentage error is:

$$errors = \frac{true - measured}{true} \times 100\%$$

$$error = \frac{|83 - 84.5|}{83} \times 100\% = 1.81\%, \text{ or } \quad error = \frac{|83 - 81.5|}{83} \times 100\% = 1.81\%$$

Or $error = \dfrac{|\pm Tolerance|}{True} \times 100\% = \dfrac{|\pm 1.5|}{83} \times 100\% = 1.81\%$

If we want to measured anther readings on the same range and determine the error, suggest we take true 60V, and 30V.

For 60V the error is:

$$error = \frac{|\pm Tolerance|}{True} \times 100\% = \frac{|\pm 1.5|}{60} \times 100\% = 2.5\%$$

And for 30V

$$error = \frac{|\pm Tolerance|}{True} \times 100\% = \frac{|\pm 1.5|}{30} \times 100\% = 5\%$$

So we can see that the error is increased as smaller voltage is measured, thus take the proper range for every measured value, the range that give big deflection on the pointer as possible.

Example (2):

(Systematic, Human errors, the difference between theoretical and practical instruments)

To measured unknown resistor by ammeter and voltmeter method. A voltmeter of sensitivity 1000Ω/V, connect in parallel with the resistor reads 100V on its 150V scale (range), while the series ammeter read 5mA. Calculate the apparent value of the resistor, actual value, and the error.

Sol.:

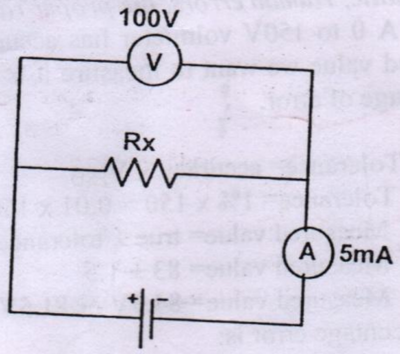
1- The apparent value of the resistor is:

$$R_{ap.} = \frac{V}{I} = \frac{100}{5mA} = 20K\Omega$$

2- The actual value of the resistor by taking the resistance of voltmeter in consider is:

$$R_V = 1000\frac{\Omega}{V} \times 150V = 150K\Omega$$

$$R_{act.} = \frac{R_{ap.} \times R_v}{R_v - R_{ap.}} = \frac{20 \times 150}{150 - 20} = 23.05K\Omega$$



3- The percent error is:

$$error = \frac{actual - apparent}{actual} \times 100\%$$

$$error = \frac{23.05 - 20}{23.05} \times 100\% = 13.22\%$$

Limiting Error

In most indicating instruments the accuracy is guaranteed to a certain percentage to a full scale reading. The limits of this deviation from the specified value are known as limiting errors or guarantee errors. For example, if the resistance of a resistor is given as 500Ω±10%, the manufacture guarantees that the resistance full between the limits 450Ω and 550Ω.

Estimation of Random Errors

These errors are due to unknown causes and occur even when all systematic errors have been accounted for. In well designed experiments few random errors usually occur, but they become important in high accuracy work. The only way to offset these errors is by increasing the number of readings and using statistical means to obtain the best approximation of the true value of the quantity under measurement.

Statistical Analysis of Data

To make statistical methods useful, the systematic errors should be small compared with random errors because statistical treatment can not improve the accuracy of measurement.

1- *Arithmetic Mean($\bar{X}$):*

It's the value lie in the medial number of measured variable and represents the most accurate measured value for the true value. Arithmetic mean is given by:

$$\bar{X} = \frac{\sum F_i \cdot X_i}{\sum F_i}$$

, where X_i is the reading values taken, and F_i is the number that each reading is occur in the measurements, or the frequency number of each reading.

2- *Deviation From The Mean(d_i):*

Deviation is the departure of a given reading from the mean value. It's given by:

$$d_i = X_i - \bar{X}$$

The deviation from the mean may have a positive or a negative value and the algebraic sum of all the deviation must be zero in symmetrical curve.

3- *Average Deviation(D):*

The average deviation is the sum of the ***absolute*** values of deviations divided by the number of readings

$$D = \frac{\sum |F_i \cdot d_i|}{\sum F_i}$$

where $\sum F_i = n$, and n= number of all readings

4- *Standard Deviation(σ):*

It's the root mean square deviation, and the standard deviation represents the variation of the reading from the mean value. For a finite number of reading

$$\sigma = \sqrt{\frac{\sum F_i.(d_i)^2}{n-1}}$$

5- *Variance(ν):*

It's defined as mean square standard deviation

$$\nu = \sigma^2$$

6- *Probable Error (r):*

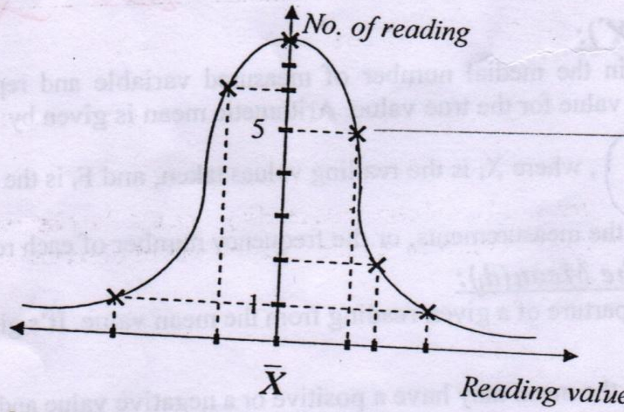
It's the maximum chance (50%) that any given measurement will have a random error no greater than ±r

$$r = \pm 0.6745\sigma$$

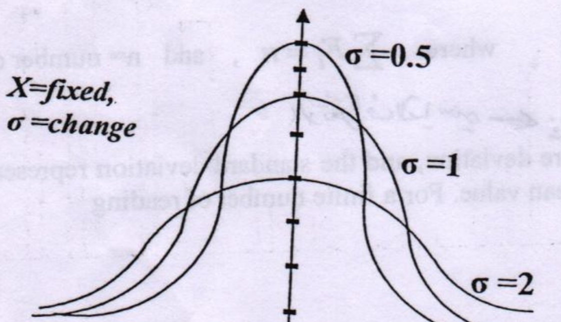
7- *Gaussian Distribution Curve:*

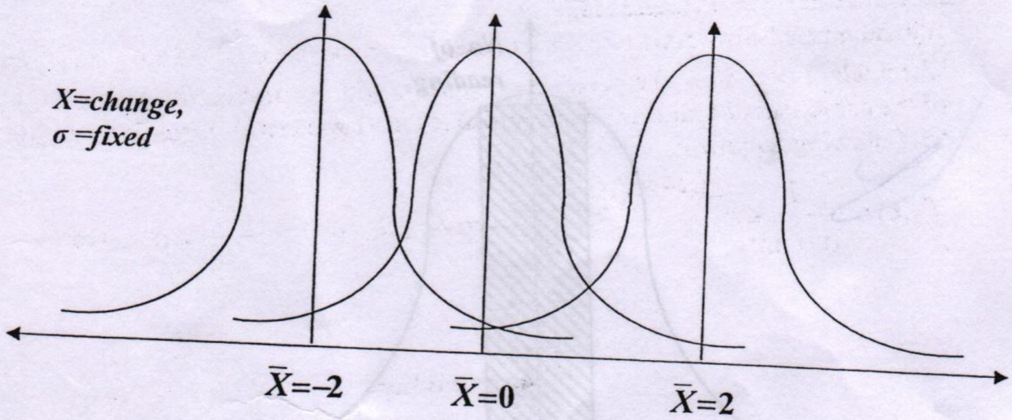
It's the normal distribution curve for random errors where $\overline{X}$ in the centre of this curve. The random errors may be positive or negative with respect to $\overline{X}$ thus lie at the two side of the curve, small errors are more probable than large errors. Gaussian curve is drawn by the following equation:

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-(X-\overline{X})^2/2\sigma^2} \qquad -\infty < X < +\infty \qquad \infty < \sigma < +\infty$$



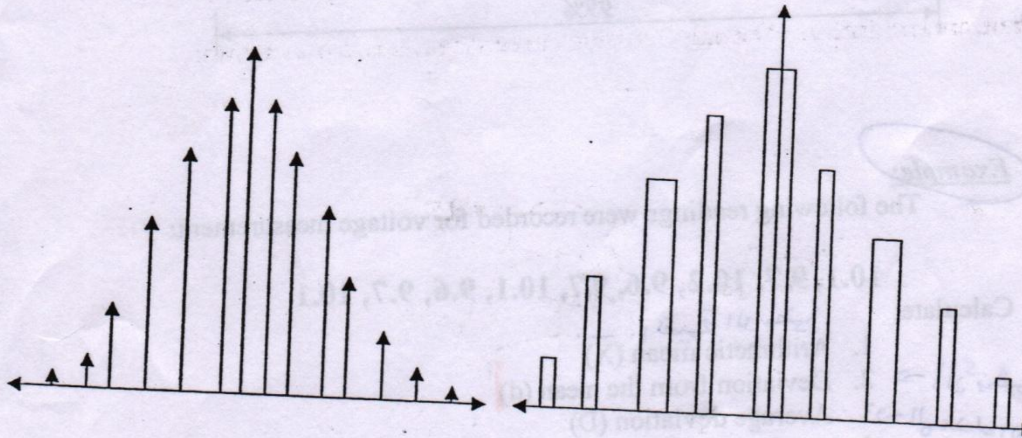
There are two factors effecting Gaussian curve shape $\overline{X}$ and σ as shown:

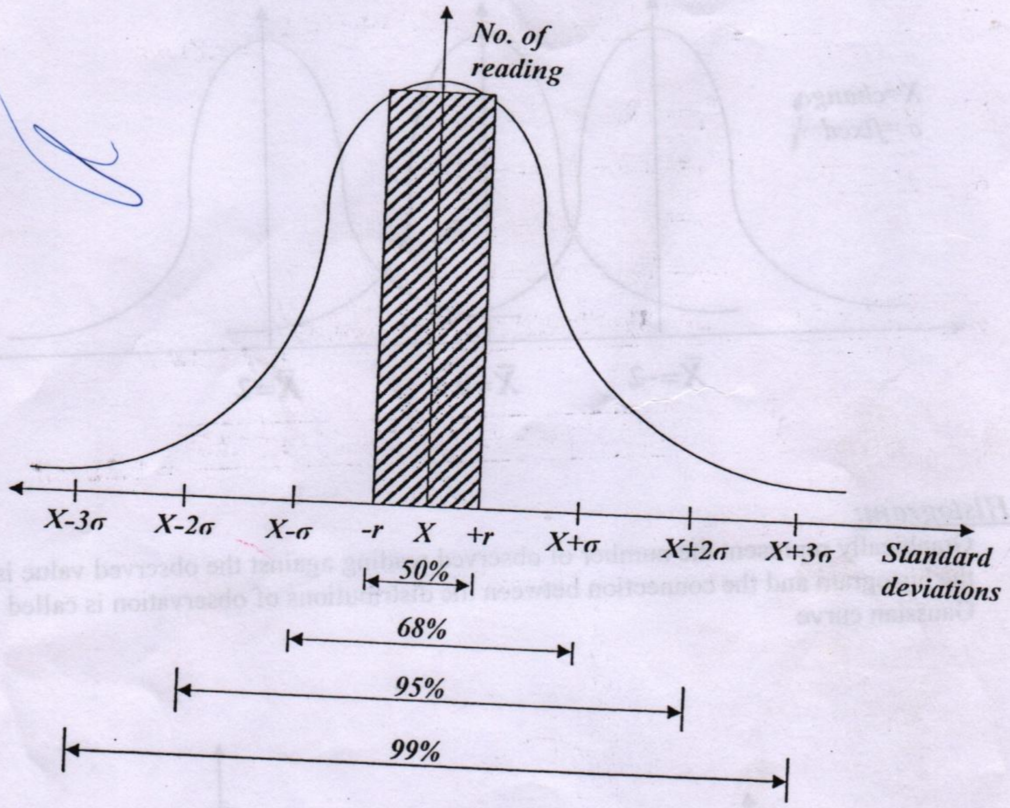




8- *Histogram:*

Graphically represent the number of observed reading against the observed value is called the histogram and the connection between the distributions of observation is called Gaussian curve





Example:

The following readings were recorded for voltage measurement:

10.1, 9.7, 10.2, 9.6, 9.7, 10.1, 9.6, 9.7, 10.1

Calculate:

1. Arithmetic mean ($\bar{X}$)
2. Deviation from the mean (d)
3. Average deviation (D)
4. standard deviation (σ)
5. Variance (V)
6. probable error (±r)

Sol.:

Rearrangement the reading in two columns with its frequency or (number of reading), thus

Reading values	*No. of reading*	*di(xi-$\bar{x}$)*
10.1	3	0.3
9.7	3	-0.1
9.6	2	-0.2
10.2	1	0.4

1- $\overline{X} = \dfrac{\sum F_i \cdot X_i}{\sum F_i} = \dfrac{3(10.1)+3(9.7)+2(9.6)+(10.1)}{9} = 9.8volt$

2- $d_i = X_i - \overline{X}$

$d_1 = 10.1 - 9.8 = 0.3volt$

$d_4 = 9.7 - 9.8 = -0.1volt$

$d_7 = 9.6 - 9.8 = -0.2volt$

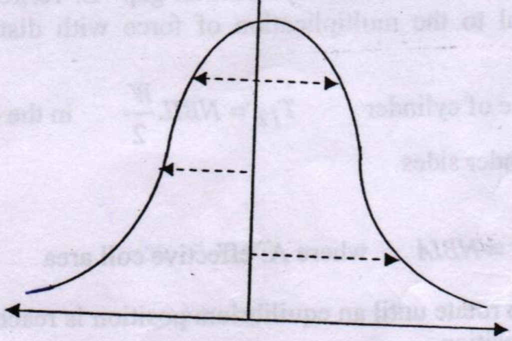
$d_9 = 10.2 - 9.8 = 0.4volt$

3- $D = \dfrac{\sum |F_i \cdot d_i|}{\sum F_i} = \dfrac{3(0.3)+3(0.1)+2(0.2)+(0.4)}{9} = 0.22volt$

4- $\sigma = \sqrt{\dfrac{\sum F_i.(d_i)^2}{n-1}} = \sqrt{\dfrac{3(0.09)+3(0.01)+2(0.04)+(0.16)}{8}} = 0.26volt$

5- $\nu = \sigma^2 = (0.26)^2 = 0.067volt^2$

6- $r = \pm 0.6745\sigma = \pm 0.6745(0.26) = \pm 0.175$



Moving Coil Instruments

There are two types of moving coil instruments namely, ***permanent magnet moving coil*** type which can only be used for ***direct*** current, voltage measurements and the ***dynamometer*** type which can be used on either ***direct or alternating*** current, voltage measurements.

Permanent Magnet Moving Coil Mechanism (PMMC)

In PMMC meter or (D'Arsonval) meter or galvanometer all are the same instrument, a coil of fine wire is suspended in a magnetic field produced by permanent magnet. According to the fundamental law of electromagnetic force, the coil will rotate in the magnetic field when it carries an electric current by electromagnetic (EM) torque effect. A pointer which attached the movable coil will deflect according to the amount of current to be measured which applied to the coil. The (EM) torque is counterbalance by the mechanical torque of control springs attached to the movable coil also. When the torques are balanced the moving coil will stopped and its angular deflection represent the amount of electrical current to be measured against a fixed reference, called a scale. If the permanent magnet field is uniform and the spring linear, then the pointer deflection is also linear.

Mathematical Representation of PMMC Mechanism

Assume there are (N) turns of wire and the coil is (L) in long by (W) in wide. The force (F) acting perpendicular to both the direction of the current flow and the direction of magnetic field is given by:

$F = N \cdot B \cdot I \cdot L$ where N: turns of wire on the coil I: current in the movable coil
B: flux density in the air gap L: vertical length of the coil

Electromagnetic torque is equal to the multiplication of force with distance to the point of suspension

$T_{I1} = NBIL\frac{W}{2}$ in one side of cylinder $T_{I2} = NBIL\frac{W}{2}$ in the other side of cylinder

The total torque for the two cylinder sides

$$T_I = 2\left(NBIL\frac{W}{2}\right) = NBILW = NBIA$$ where A: effective coil area

This torque will cause the coil to rotate until an equilibrium position is reached at an angle θ with its original orientation. At this position

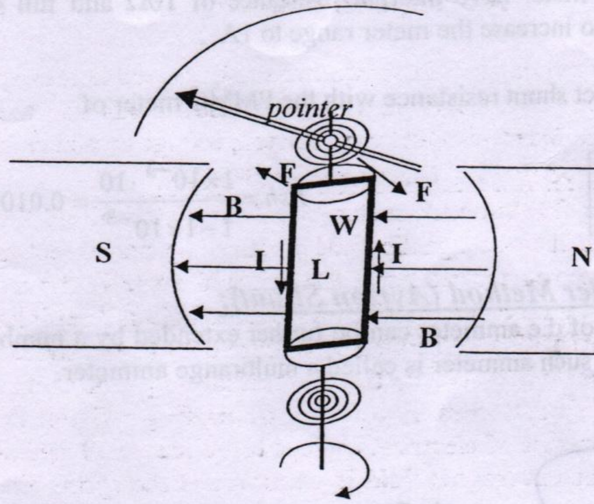
Electromagnetic torque = control spring torque

$$T_I = T_s$$

Since $T_s = K\theta$

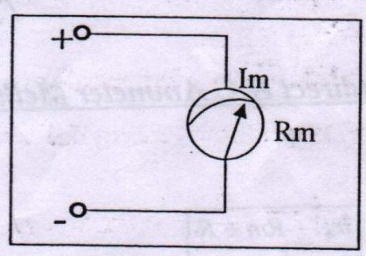
So $\theta = \frac{NBA}{K}I$ where $C = \frac{NBA}{K}$ Thus $\theta = CI$

The angular deflection proportional linearly with applied current



1- *D.c Ammeter:*

An Ammeter is always connected in series with a circuit branch and measures the current flowing in it. Most d.c ammeters employ a d'Arsonval movement, an ideal ammeter would be capable of performing the measurement without changing or distributing the current in the branch but real ammeters would possess some internal resistance.



Extension of Ammeter Range:

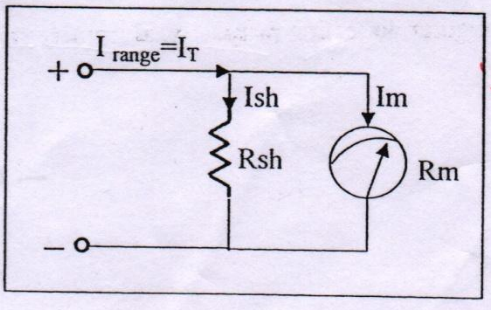
Since the coil winding in PMMC meter is *small and light*, they can carry only small currents (μA-1mA). Measurement of large current requires ***a shunt external resistor*** to connect with the meter movement, so only a fraction of the total current will passes through the meter.

$Vm = Vsh$

$\text{Im}\, Rm = Ish Rsh$

$Ish = I_T - \text{Im}$

$$Rsh = \frac{\text{Im}\, Rm}{I_T - \text{Im}}$$



Example:

If PMMC meter have internal resistance of 10Ω and full scale range of 1mA. Assume we wish to increase the meter range to 1A.

Sol.

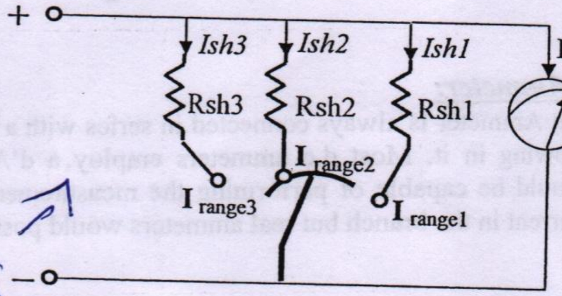
So we must connect shunt resistance with the PMMC meter of

$$Rsh = \frac{\text{Im}\, Rm}{I_T - \text{Im}} \qquad Rsh = \frac{1\times10^{-3}\cdot 10}{1-1\times10^{-3}} = 0.01001\Omega$$

a) *Direct D.c Ammeter Method (Ayrton Shunt):*

The current range of d.c ammeter can be further extended by a number of shunts selected by a range switch; such ammeter is called a multirange ammeter.

$$Rsh_* = \frac{\text{Im}\, Rm}{Ir_* - \text{Im}}$$

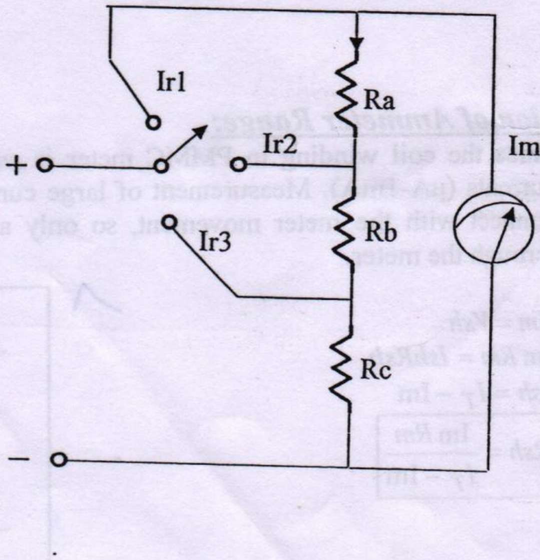


b) *Indirect D.C Ammeter Method:*

$$\frac{Ir_*}{\text{Im}} = \frac{Rm + R}{r_*}$$

Where R=Ra+ Rb+ Rc

And r = parallel resistors branch with the meter



Example (1):

Design a multirange ammeter by using ***direct method*** to give the following ranges 10mA, 100mA, 1A, 10A, and 100A. If d'Arsonval meter have internal resistance of 10Ω and full scale current of 1mA.

Sol:

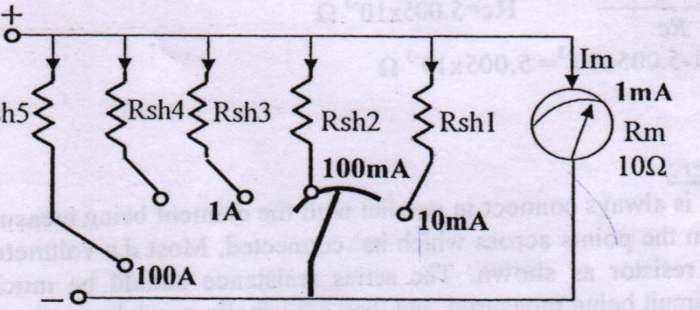
Rm=10Ω Im=1mA

$$Rsh_* = \frac{Im\ Rm}{Ir_* - Im}$$

$$Rsh1 = \frac{1\times10^{-3}\cdot10}{(10-1)\times10^{-3}} = 1.11\Omega$$

$$Rsh2 = \frac{1\times10^{-3}\cdot10}{(100-1)\times10^{-3}} = 0.101\Omega \qquad Rsh3 = \frac{1\times10^{-3}\cdot10}{1-1\times10^{-3}} = 0.0101\Omega$$

$$Rsh4 = \frac{1\times10^{-3}\cdot10}{10-1\times10^{-3}} = 0.0011\Omega \qquad Rsh5 = \frac{1\times10^{-3}\cdot10}{100-1\times10^{-3}} = 0.00011\Omega$$



Example (2):

Design an Ayrton shunt by ***indirect method*** to provide an ammeter with current ranges 1A, 5A, and 10A, if PMMC meter have internal resistance of 50Ω and full scale current of 1mA.

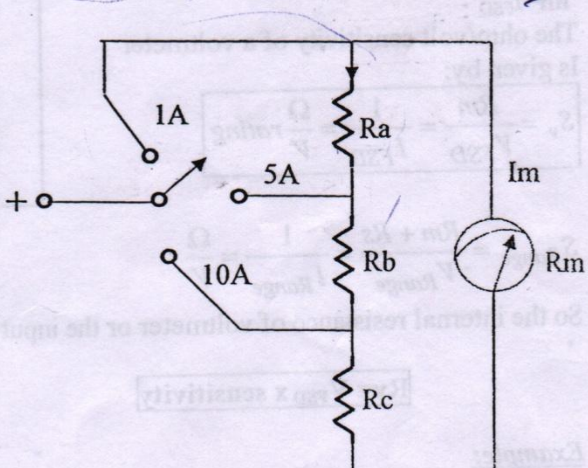
Sol.:

Rm=50Ω I_{FSD}=Im=1mA

$$\frac{Ir_*}{Im} = \frac{Rm + R}{r_*}$$

Where R=Ra+ Rb+ Rc

And r = parallel resistors branch with the meter



1- *For 1A Range:*

$$\frac{I1}{Im} = \frac{Rm + R}{R}$$

$$\frac{1A}{1mA}=\frac{50+R}{R} \qquad R=0.05005\Omega$$

2- *For 5A Range:*

$$\frac{I2}{\text{Im}}=\frac{Rm+R}{Rb+Rc} \qquad r=Rb+Rc$$

$$\frac{5A}{1mA}=\frac{50+0.05005}{Rb+Rc} \qquad Rb+Rc=0.01001\Omega$$

Ra=R-(Rb+Rc) **Ra=0.05-0.01001=0.04004 Ω**

3- *For 10A Range:*

$$\frac{I3}{\text{Im}}=\frac{Rm+R}{Rc} \qquad r=Rc$$

$$\frac{10A}{1mA}=\frac{50+0.05005}{Rc} \qquad \mathbf{Rc}=5.005\times10^{-3}\ \Omega$$

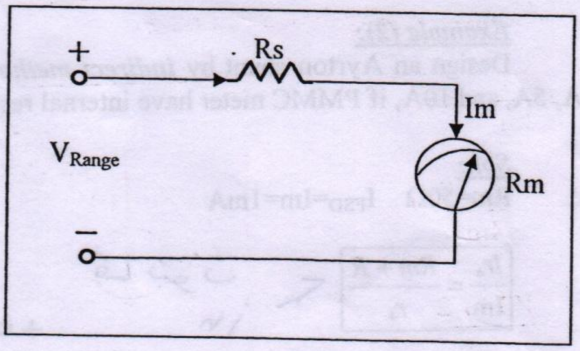
Rb$=0.01001-5.005\times10^{-3}=5.005\times10^{-3}\ \Omega$

2- D.C Voltmeter:

A voltmeter is always connect in parallel with the element being measured, and measures the voltage between the points across which its' connected. Most d.c voltmeter employ PMMC meter with series resistor as shown. The series resistance should be much larger than the impedance of the circuit being measured, and they are usually much larger than Rm.

$$Rs=R_T-Rm$$

$$Rs=\frac{V_{range}}{\text{Im}}-Rm$$



$Im=I_{FSD}$

The ohm/volt sensitivity of a voltmeter Is given by:

$$S_v=\frac{Rm}{V_{FSD}}=\frac{1}{I_{FSD}}=\frac{\Omega}{V}\,rating$$

$$S_{Range}=\frac{Rm+Rs}{V_{Range}}=\frac{1}{I_{Range}}=\frac{\Omega}{V}$$

So the internal resistance of voltmeter or the input resistance of voltmeter is

$Rv=V_{FSD}\times$ sensitivity

Example:

We have a micro ammeter and we wish to adapted it so as to measure 1volt full scale, the meter has internal resistance of 100Ω and I_{FSD} of 100μA.

Sol.:

$$Rs = \frac{V}{Im} - Rm \qquad Rs = \frac{1}{0.0001} - 100 = 9900\Omega = 9.9K\Omega$$

So we connect with PMMC meter a series resistance of 9.9KΩ to convert it to voltmeter

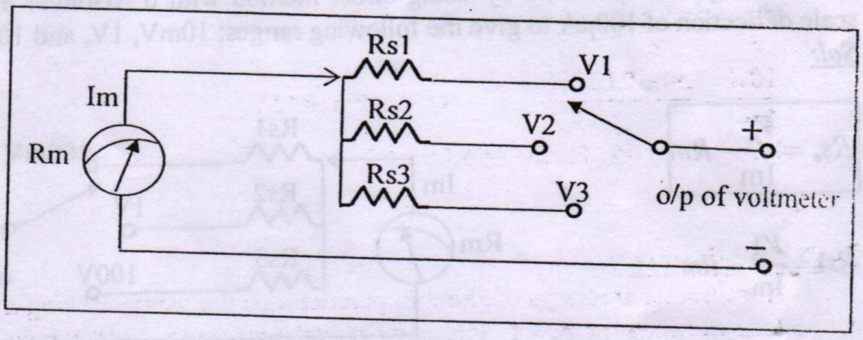
Extension of Voltmeter Range:

Voltage range of d.c voltmeter can be further extended by a number of series resistance selected by a range switch; such a voltmeter is called multirange voltmeter.

a) *Direct D.c Voltmeter Method:*

In this method each series resistance of multirange voltmeter is connected in direct with PMMC meter to give the desired range.

$$Rs_{*} = \frac{V_{*}}{Im} - Rm$$



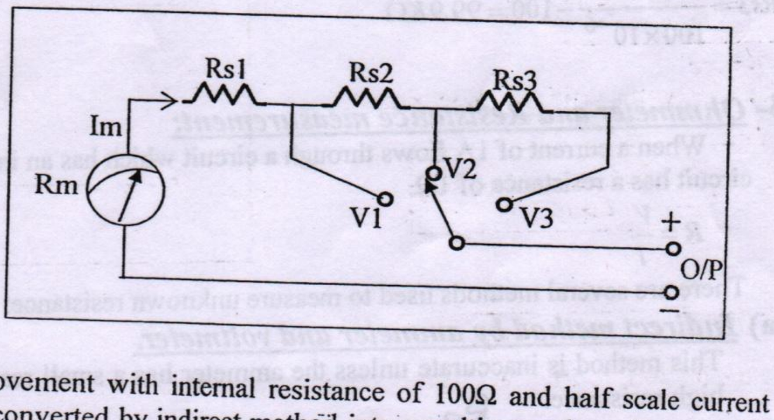
b) *Indirect D.c Voltmeter Method:*

In this method one or more series resistances of multirange voltmeter is connected with PMMC meter to give the desired range.

$$Rs1 = \frac{V1}{Im} - Rm$$

$$Rs2 = \frac{V2 - V1}{Im}$$

$$Rs3 = \frac{V3 - V2}{Im}$$



Example (1):

A basic d'Arsonval movement with internal resistance of 100Ω and half scale current deflection of 0.5 mA is to be converted by indirect method into a multirange d.c voltmeter with voltages ranges of 10V, 50V, 250V, and 500V.

Sol:

$I_{FSD} = I_{HSD} \times 2$

$I_{FSD} = 0.5mA \times 2 = 1mA$

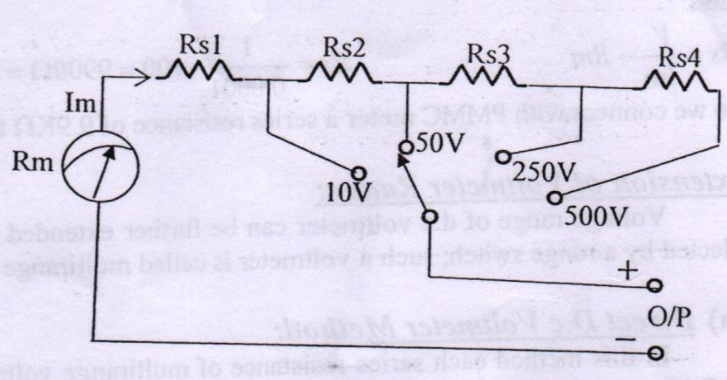
$$Rs1 = \frac{V1}{Im} - Rm \qquad Rs1 = \frac{10}{1mA} - 100 = 9.9K\Omega$$

$$Rs2 = \frac{V2 - V1}{\text{Im}}$$

$$Rs2 = \frac{50 - 10}{1 \times 10^{-3}} = 40K\Omega$$

$$Rs3 = \frac{250 - 50}{1 \times 10^{-3}} = 200K\Omega$$

$$Rs4 = \frac{500 - 250}{1 \times 10^{-3}} = 250K\Omega$$



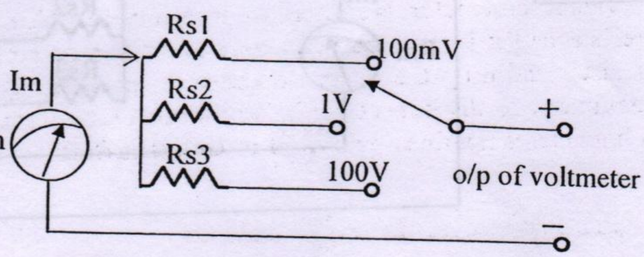
Example (2):

Design d.c voltmeter by using direct method with d'Arsonval meter of 100Ω and full scale deflection of 100μA to give the following ranges: 10mV, 1V, and 100V.

Sol:

$$\boxed{Rs_* = \frac{V_*}{\text{Im}} - Rm}$$

$$Rs1 = \frac{V1}{\text{Im}} - Rm$$



$$Rs1 = \frac{10mV}{100\mu A} - 100 = 0\Omega$$

$$Rs2 = \frac{1}{100 \times 10^{-6}} - 100 = 9.9K\Omega$$

$$Rs3 = \frac{100}{100 \times 10^{-6}} - 100 = 99.9K\Omega$$

3- Ohmmeter and Resistance measurement:

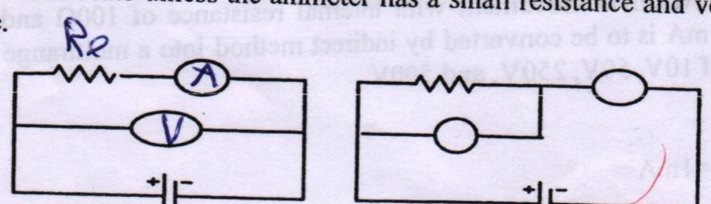
When a current of 1A flows through a circuit which has an impressed voltage of 1volt, the circuit has a resistance of 1Ω.

$$R = \frac{V}{I}$$

There are several methods used to measure unknown resistance:

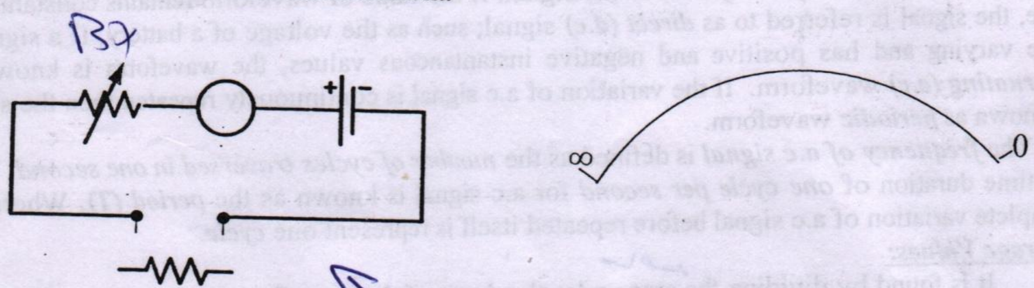
a) *Indirect method by ammeter and voltmeter.*

This method is inaccurate unless the ammeter has a small resistance and voltmeter have a high resistance.



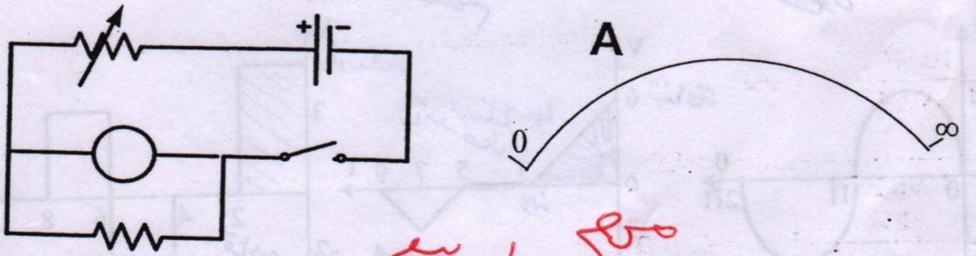
b) *Series Ohmmeter:*

Rx is the unknown resistor to be measured, R2 is variable adjusted resistance so that the pointer read zero at short circuit test. The scale of series ohmmeter is nonlinear with zero at the right and infinity at extreme left. Series ohmmeter is the most generally used meter for resistance measurement.



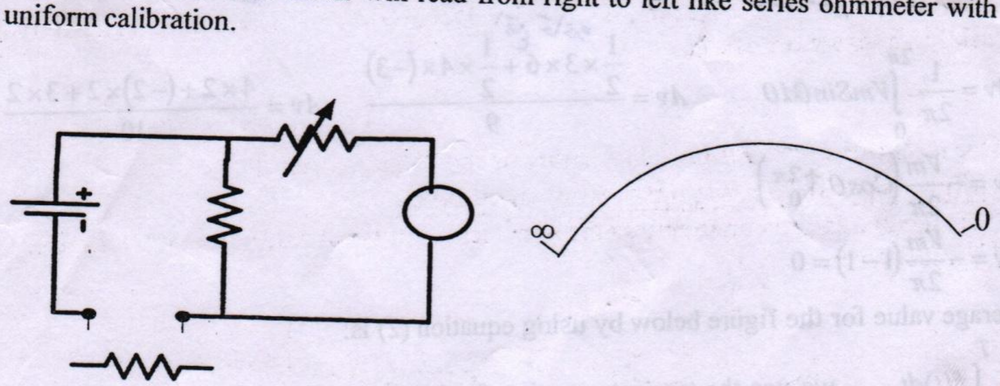
c) *Shunt Ohmmeter:*

Shunt ohmmeter are used to measure very low resistance values. The unknown resistance Rx is now shunted across the meter, so portion of current will pass across this resistor and drop the meter deflection proportionately. The switch is necessary in shunt ohmmeter to disconnect the battery when the instrument is not used. The scale of shunt ohmmeter is nonlinear with zero at the left and infinity at extreme right.



d) *Voltage Divider (potentiometer):*

The meter of voltage divider is voltmeter that reads voltage drop across Rs which dependent on Rx. This meter will read from right to left like series ohmmeter with more uniform calibration.



A.c Measuring Instrument

Review on Alternating Signal:

The instantaneous values of electrical signals can be graphed as they vary with time. Such graphs are known as the ***waveforms*** of the signal. If the value of waveform remains constant with time, the signal is referred to as ***direct (d.c)*** signal; such as the voltage of a battery. If a signal is time varying and has positive and negative instantaneous values, the waveform is known as ***alternating (a.c)*** waveform. If the variation of a.c signal is continuously repeated then the signal is known as ***periodic*** waveform.

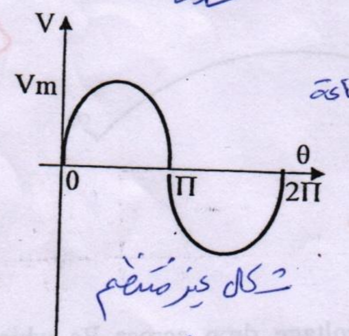
The ***frequency of a.c signal*** is defined as the *number of cycles traversed in one second*. Thus the time duration of *one cycle per second* for a.c signal is known as the ***period (T)***. Where the complete variation of a.c signal before repeated itself is represent one *cycle*.

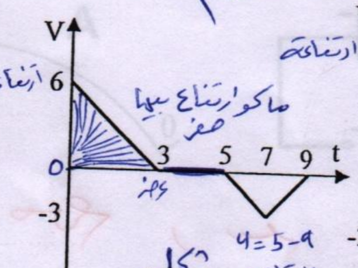
Average Values:

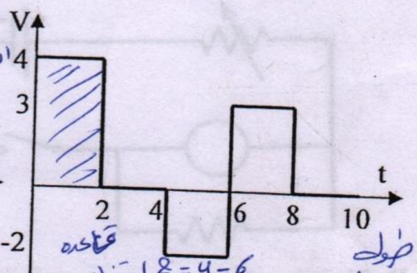
It is found by dividing the area under the curve of the waveform in one period (T) by the time of the period.

Average value= $\dfrac{\text{Algebraic sum of the areas under the curve}}{\text{Length of the curve}}$

$$Av = \frac{\sum areas}{T} \quad \ldots\ldots\ldots\ldots (1) \qquad \text{or} \qquad Av = \frac{1}{T}\int_0^T f(t)dt \quad \ldots\ldots\ldots\ldots (2)$$







$$Av = \frac{1}{2\pi}\int_0^{2\pi} VmSin\theta d\theta \qquad Av = \frac{\frac{1}{2}\times 3\times 6 + \frac{1}{2}\times 4\times(-3)}{9} \qquad Av = \frac{4\times 2 + (-2)\times 2 + 3\times 2}{10}$$

$$Av = -\frac{Vm}{2\pi}\left(Cos\theta \uparrow_0^{2\pi}\right)$$

$$Av = -\frac{Vm}{2\pi}(1-1) = 0$$

The average value for the figure below by using equation (2) is:

$Av = \frac{1}{T}\int_0^T f(t)dt$ we use the tangent equation for $(x_o,y_o)=(0,0)$, and $(x_1,y_1)=(3,6)$ to find the function of f(t)

$AV = \frac{1}{T}\int_0^{\pi} f\,As\,d$

$\frac{1}{2\pi}\int_0^{\pi} vm\sin\theta\, d\theta \Rightarrow \frac{vm}{2\pi}\int_0^{\pi}\sin\theta\, d\theta \Rightarrow \frac{vm}{2\pi}\left[-\cos\theta\right]_0^{\pi}$

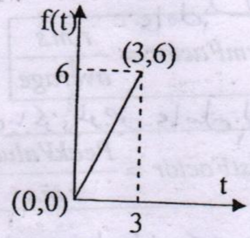
$\frac{vm}{2\pi}\left[-\cos(\pi) - 1 - \cos(0)\right] = \frac{vm}{\pi}$

معادلة الخط المستقيم

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1} \rightarrow \frac{y-0}{x-0}=\frac{6-0}{3-0} \Rightarrow \frac{y}{x}=\frac{6}{3}=2 \Rightarrow\Rightarrow y=2x$$

$\boxed{f(t)=2t}$

$$Av=\frac{1}{3}\int_0^3 (2t)dt$$



$$Av=\frac{2}{3}\left(\frac{t^2}{2}\uparrow_0^3\right) \qquad Av=\frac{1}{3}\left((3)^2-(0)^2\right)=\frac{9}{3}=3$$

جذر متوسط تربيع — عندما يكون التربيع

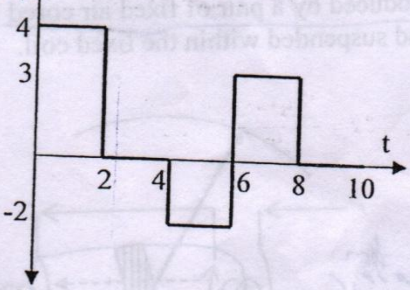
Root Mean Square Value(effective value of a.c signal):

لا شكل بالمنتظم - الغير منتظم - مربع، مثلث

The r.m.s value of a waveform refers to its power capability. It is refer to the effective value of a.c signal because the r.m.s value equal to the value of a d.c signal which would deliver the same power if it replaced with a.c signal.

$$r.m.s=\sqrt{\frac{\sum area(V)^2}{T}}$$ *(for square waveform only)*

(اربعة) عن الـ y اكبر f of f(t)

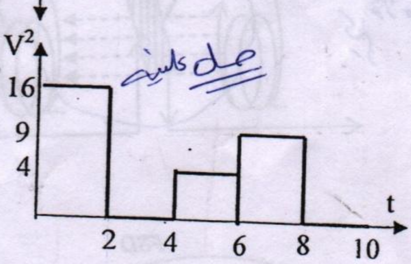


1- $r.m.s=\sqrt{\frac{16\times2+4\times2+9\times2}{10}}$

اي الشكل يعيد يوصلها الى 4

In general form the r.m.s value has the following aqua.

r.m.s= √Average $f(t)^2$

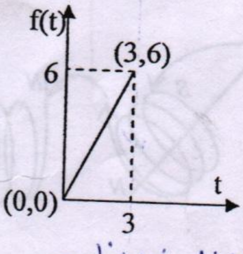


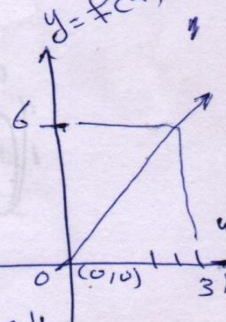
حل عليه

$$\boxed{r.m.s=\sqrt{\frac{1}{T}\int_0^T f(t)^2dt}}$$

2- If f(t) = 2t then its r.m.s value is:

$$r.m.s=\sqrt{\frac{1}{3}\int_0^3 (2t)^2dt}$$





$$r.m.s=\sqrt{\frac{4}{3}\left(\frac{t^3}{3}\uparrow_0^3\right)}=\sqrt{\frac{4}{9}\left((3)^3-(0)^3\right)}=\sqrt{\frac{4\times27}{9}}=3.46$$

3- If f(t) = Vm Sinθdθ

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$$r.m.s=\sqrt{\frac{1}{2\pi}\int_0^{2\pi}Vm^2Sin^2\theta d\theta} \qquad r.m.s=\sqrt{\frac{Vm^2}{2\pi}\int_0^{2\pi}\frac{1-Cos2\theta}{2}d\theta}$$

$$r.m.s=\left\{\frac{Vm^2}{4\pi}\left[\int_0^{2\pi}d\theta-\int_0^{2\pi}Cos2\theta d\theta\right]\right\}^{\frac{1}{2}} \qquad r.m.s=\sqrt{\frac{Vm^2}{4\pi}\left[\theta\Big|_0^{2\pi}-\frac{1}{2}Sin2\theta\Big|_0^{2\pi}\right]}$$

Q/ calculate Avarge value of pair (0,0), (3,6)

$AV = \frac{1}{T}\int_0^T f(t)\,dt$

$\frac{y-0}{x-0} = \frac{6-0}{3-0} \Rightarrow \frac{y}{x} = \frac{6}{3} = 2 \Rightarrow y = 2x$

$f(t) = 2t$

$AV = \frac{1}{3}\int_0^3 (2t)\,dt$

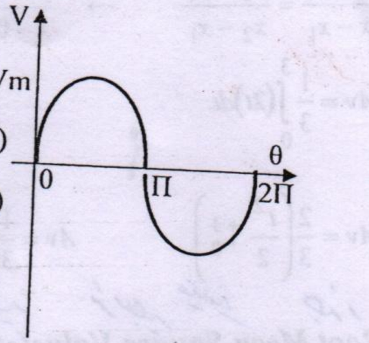
$\frac{2}{3}\int_0^3 t\,dt \Rightarrow \frac{2}{3}\left|\frac{t^2}{2}\right|_0^3$

$\frac{1}{3}\left[(3)^2-(0)^2\right] = \frac{1}{3}\times 9 = 3$

Q/calcalue (R m s) for pair (0,0) (3,6)

$F(t) = 2 + \frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$ (0 → 3)

$$r.m.s = \sqrt{\frac{Vm^2}{4\pi}[2\pi - 0]} = \sqrt{\frac{Vm^2}{2}} = \frac{Vm}{\sqrt{2}}$$



$$FormFactor = \frac{r.m.s}{average}$$ for Sine wave F.F=1.11 (F.W.R)

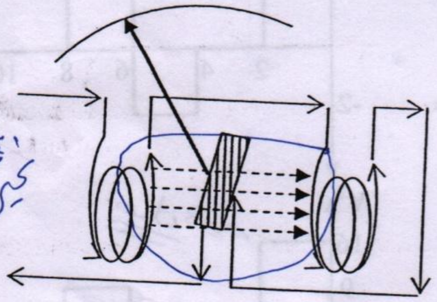
F.F=1.57 (H.W.R)

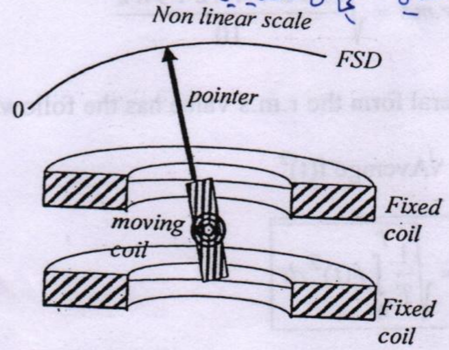
$$CrestFactor = \frac{PeakValue}{r.m.s}$$

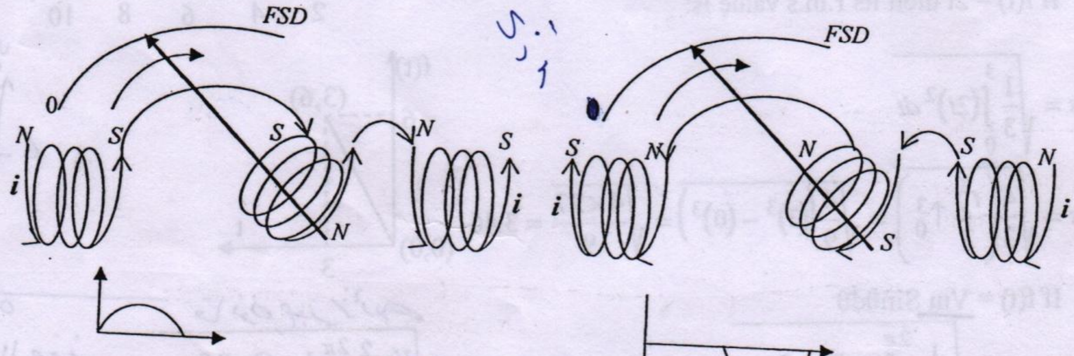
Rms value → Avarge

Dynamometer:

This instrument is suitable for the measurement of direct and alternating current, voltage and power. The deflecting torque in dynamometer is relies by the interaction of magnetic field produced by a pair of fixed air cored coils and a third air cored coil capable of angular movement and suspended within the fixed coil.





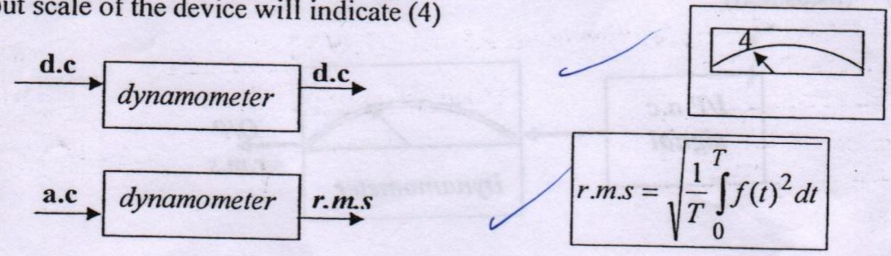


$T_i = N\vec{B}i_m A$, $\vec{B} \alpha i_f$ thus $T_i \alpha i_m i_f A \Rightarrow\Rightarrow$ so $T_i \alpha i^2$

$\theta\ \alpha$ average i^2 , since $r.m.s = \sqrt{average f(t)^2}$

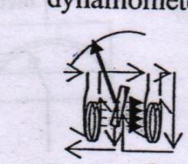
The ***output scale is calibrated to give the r.m.s value of a.c signal*** by taking the square roots of the inside measured value.

O/P scale $= r.m.s = \sqrt{average(i)^2}$, for example if (average i^2) = 16 inside the measuring device, the output scale of the device will indicate (4)



Meters

- d.c meters measure d.c or Av values
  - PMMC
  - dynamometer
- a.c meters measure r.m.s values
  - Average Responding a.c meter ⇓ Rectifier + PMMC
  - True Responding a.c meter ⇓ dynamometer

PMMC	dynamometer
	
T α i	T α i^2
B constant	B varied
linear scale	non linear scale
θα i	θα average i^2

1-*Average Responding a.c Meter:*

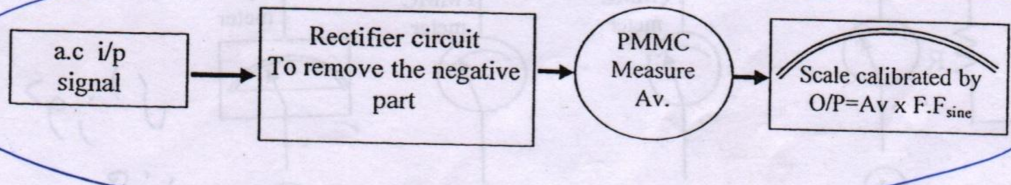
O/P (r.m.s) = Av x $F.F_{sine\ wave}$ (measured)

$F.F_{sine\ wave}$(F.W.R) = 1.11

$F.F_{sine\ wave}$ (H.W.R) = 1.57

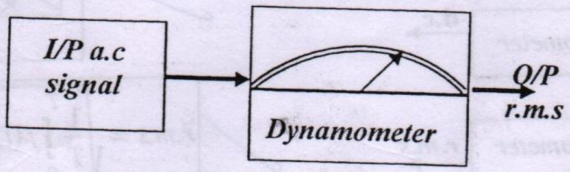
O/P (r.m.s) = Av x $F.F_{true}$ (true)

$F.F_{true}$ = The form factor of any input signal (sine, square, or any thing)



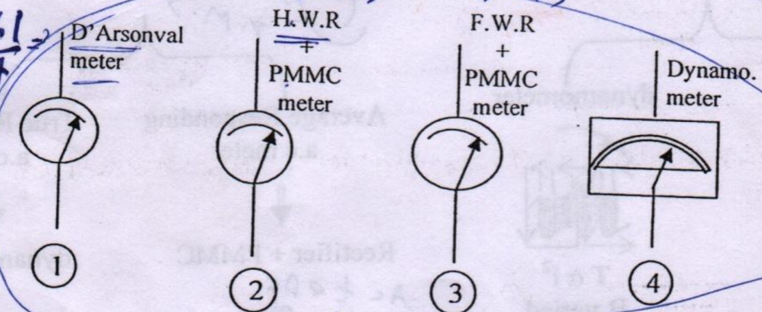
2- *True Responding a.c Meter (Dynamometer):*

O/P (r.m.s) = Av x $F.F_{true}$ $F.F_{true}$ = The form factor of any input signal
(true) = (measured)



Example:

What will be the out put of the following meters, if an average responding a.c meter of half-wave rectifier read (4.71v), and true form factor of input waveform is (1.414).



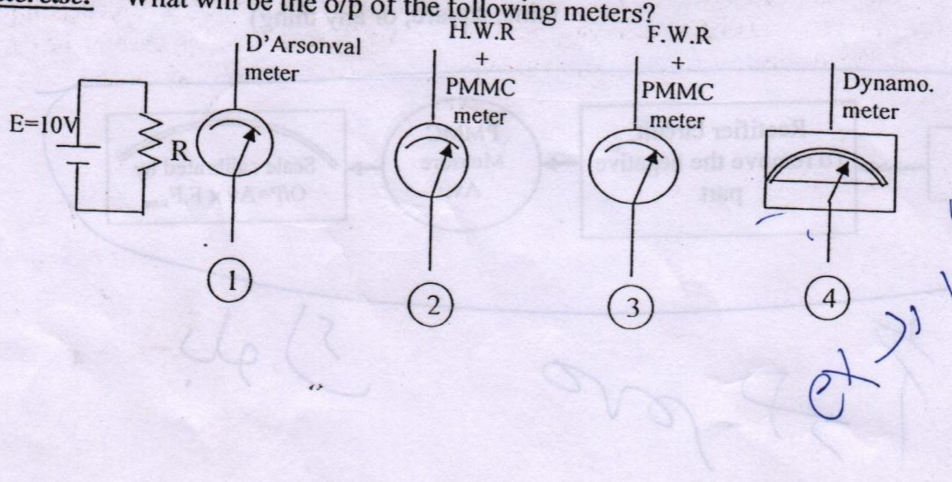
Sol:

$r.m.s_{measured} = 1.57 \times Av$ for average responding a.c meter of half wave rectifier

$4.71 = 1.57 \times Av$ $\Rightarrow Av = \dfrac{4.71}{1.57} = 3V$

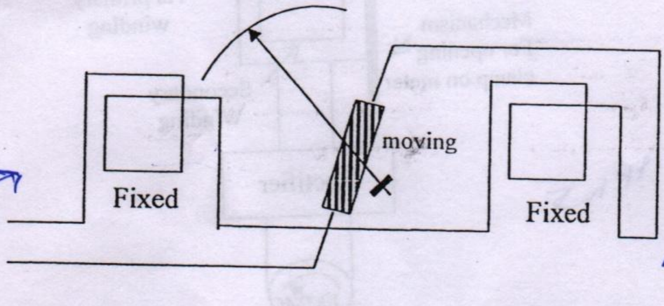
1. D'Arsonval meter read Av = 3V
2. HWR+PMMC (*Average responding of halve wave rectifier*) meter = 4.71V
3. FWR+PMMC (*Average responding of full wave rectifier*) meter = 1.11 x 3 = 3.33V
4. Dynamometer = $F.F_{(true)}$ x Av
   $r.m.s_{(true)}$ = 1.414 x 3 = 4.242V

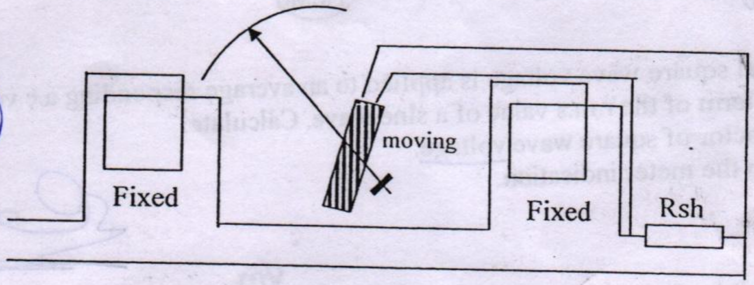
Exercise: What will be the o/p of the following meters?



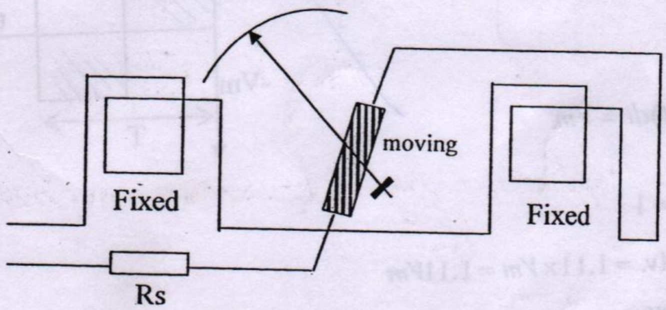
Dynamometer As Ammeter And Voltmeter:

For small current measurement (5mA to 100mA), fixed and moving coils are connect in series. While larger current measurement (up to 20A) , the moving coil is shunted by a small resistance.





To convert such an instrument to a voltmeter only a rather big series resistance is connected with the moving coil.



Clamp on Meters (Average Responding A.C meter):

One application of average responding a.c meters is the ***clamp on meter*** which is used to measured a.c current, voltage in a wire ***with out having to break*** the circuit being measured. The meter having use the transformer principle to detect the current. That is, the clamp on device of the meter serves as the core of a transformer. The current carrying wire is the primary winding of the transformer, while the secondary winding is in the meter. The alternating current in the primary is coupled to the secondary winding by the core, and after being rectified the current is sensed by a d'Arsonval meter.

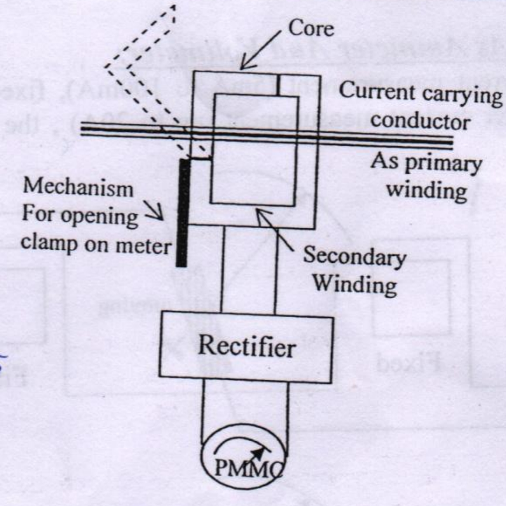
(1) $AV = \frac{4.71}{1.57} = 3V$

(2) $1.57 \times AV = 1.57 \times 3$

(3) $1.11 \times AV = 1.11 \times 3$

(4) f.f true × AV

$1.417 \times 3 =$

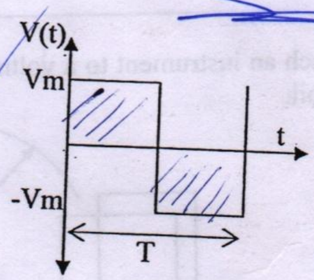


Example:

The symmetrical square wave voltage is applied to an average responding a.c voltmeter with a scale calibrated in term of the r.m.s value of a sine wave. Calculate:

1. The form factor of square wave voltage.
2. The error in the meter indication.

Sol:



$$Vrms_{(True)} = \sqrt{\frac{1}{T}\int_{o}^{T} V(t)^2\,dt} = Vm$$

$$Vaverage_{(True)} = \frac{2}{T}\int_{o}^{\frac{T}{2}} V(t)dt = Vm$$

$$F.F_{(True)} = \frac{Vrms}{Vav.} = \frac{Vm}{Vm} = 1$$

$$Vrms_{(measured)} = 1.11 \times Av. = 1.11 \times Vm = 1.11Vm$$

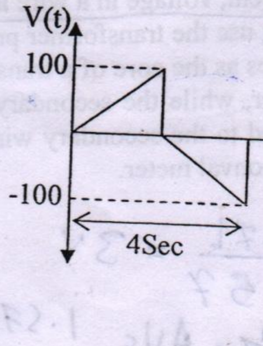
$$Error = \frac{Vrms_{(True)} - Vrms_{(measured)}}{Vrms_{(True)}} \times 100\%$$

$$Error = \frac{Vm - 1.11Vm}{Vm} \times 100\%$$

Exer.:

Repeat the above example for saw tooth waveform shown

Sol:



$V(t)=25t$

$Vav.=50V$

$Vrms_{(True)}=57.75V$

$Vrms_{(Measured)}=55.5V$

$F.F_{(True)}=1.154$

$Error=0.0389\%$

Bridges and Their Application

Bridge circuit are extensively used for ***measuring component values***, such as *resistance, inductance, capacitance*, and other circuit parameters directly derived from component values such as *frequency, phase angle, and temperature*. Bridge accuracy measurements are very high because their circuit merely compares the value of an unknown component to that of an accurately known component (a standard).

1- ***D.c Bridges:***

The basic d.c bridges consist of four resistive arms with a source of emf (a battery) and a null detector usually galvanometer or other sensitive current meter. D.c bridges are generally used for the measurement of resistance values.

a) ***Wheatstone Bridge:***

This is the best and commonest method of measuring ***medium*** resistance values in the range of 1Ω to the low megohm. The current through the galvanometer depends on potential difference between point (c) and (d). The ***bridge*** is said to be ***balance*** when potential difference across the galvanometer is zero volts, so there is no current through the galvanometer (***Ig=0***). This condition occurs when Vca=Vda or Vcb=Vdb hence the bridge is balance when

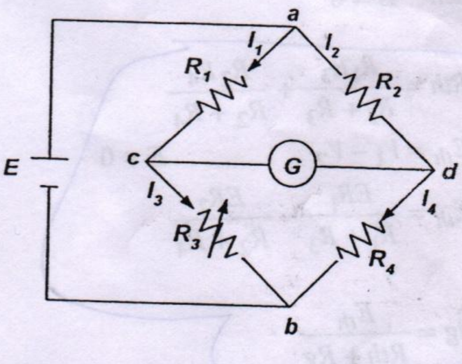
$V1 = V2$ …….. (1) Since $I_g = 0$ so by voltage divider rule

$V_1 = E\dfrac{R_1}{R_1 + R_3}$ ….. (2) and

$V_2 = E\dfrac{R_2}{R_2 + R_4}$ ….. (3)

Substitute equations (2) & (3) in equ. (1)

$$\frac{R_1}{R_1 + R_3} = \frac{R_2}{R_2 + R_4}$$



Thus $\boxed{R_1 R_4 = R_2 R_3}$ is the balance equation for Wheatstone bridge

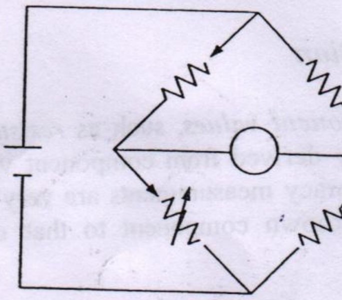
So, if three of resistance values are known, the fourth unknown ones can be determined.

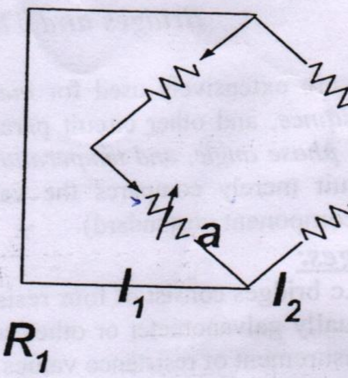
$$\boxed{R_4 = \frac{R_3 R_2}{R_1}}$$

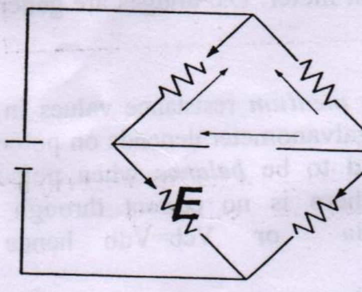
R_3 are called the standard arm of the bridge and resistors R_2 and R_1 are called the ratio arms.

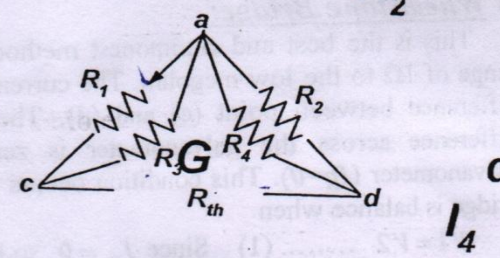
Thevenin Equivalent Circuit:

To determine whether or not the galvanometer has the required sensitivity to detect an unbalance condition, it is necessary to calculate the galvanometer current for small unbalance condition. The solution is approached by converting the Wheatstone bridge to its thevenin equivalent. Since we are interested in the current through the galvanometer, the thevenin equivalent circuit is determined by looking into galvanometer terminals (c) and (d).









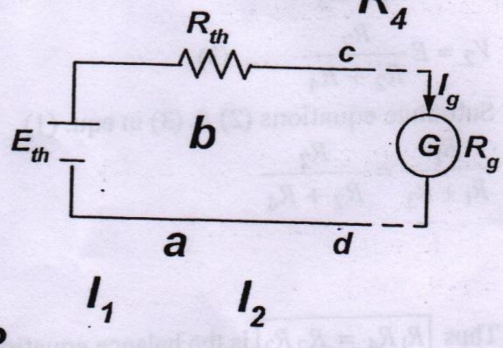
when $E = 0$

$$Rth = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4}$$

$$E_{th} = V_1 - V_2 \qquad E \neq 0$$

$$E_{th} = \frac{ER_1}{R_1 + R_3} - \frac{ER_2}{R_2 + R_4}$$

$$Ig = \frac{E_{th}}{Rth + Rg}$$



and galvanometer deflection (d) is: **$d = Ig$ x current sensitivity (mm/μA)**

b) Kelvin Bridge:

Kelvin bridge is a modification of the Wheatstone bridge and provides greatly increased accuracy in the measurement of ***low value*** resistance, generally below (1Ω). It is eliminate errors due to contact and leads resistance. (R_y) represent the resistance of the connecting lead from R_3 to R_4. Two galvanometer connections are possible, to point (m) or to point (n).

1- ***If the galvanometer connect to point (m) then***

$R_4 = R_x + R_y$ therefore unknown resistance will be higher than its actual value by R_y

2- ***If the galvanometer connect to point (n) then***

$R_4 = R_3 + R_y$ therefore unknown resistance will be lower than its actual value by R_y

3- ***If the galvanometer connect to point (p) such that***

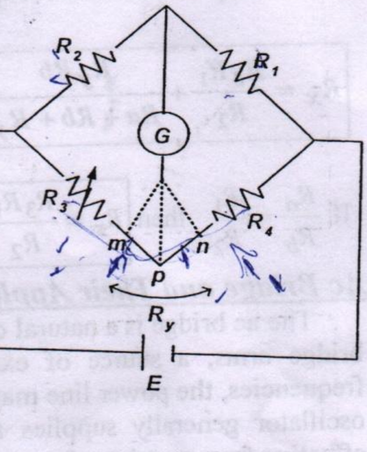
$$\frac{R_{np}}{R_{mp}} = \frac{R_1}{R_2} \quad \ldots\ldots\ldots (1)$$

At balance condition

$$R_2\left(R_x + R_{np}\right) = R_1\left(R_3 + R_{mp}\right) \quad \ldots\ldots (2)$$

Substituting equ.(1) in to equ.(2) we obtain

$$R_x + \left(\frac{R_1}{R_1 + R_2}\right)R_y = \frac{R_1}{R_2}\left[R_3 + \left(\frac{R_2}{R_1 + R_2}\right)R_y\right]$$



This reduces to $\boxed{R_x = \frac{R_1}{R_2} R_3}$

So the effect of the resistance of the connecting lead from point (**m**) to point (**n**) has be eliminated by connecting the galvanometer to the intermediate position (**p**).

c) Kelvin Double Bridge:

Kelvin double bridge is used for measuring ***very low*** resistance values from approximately (1Ω to as low as $1\times10^{-5}\Omega$). The term double bridge is used because the circuit contains a second set of ratio arms labelled Ra and Rb. If the galvanometer is connect to point (**p**) to eliminates the effect of (yoke resistance **Ry**).

$$\frac{R_a}{R_b} = \frac{R_1}{R_2}$$

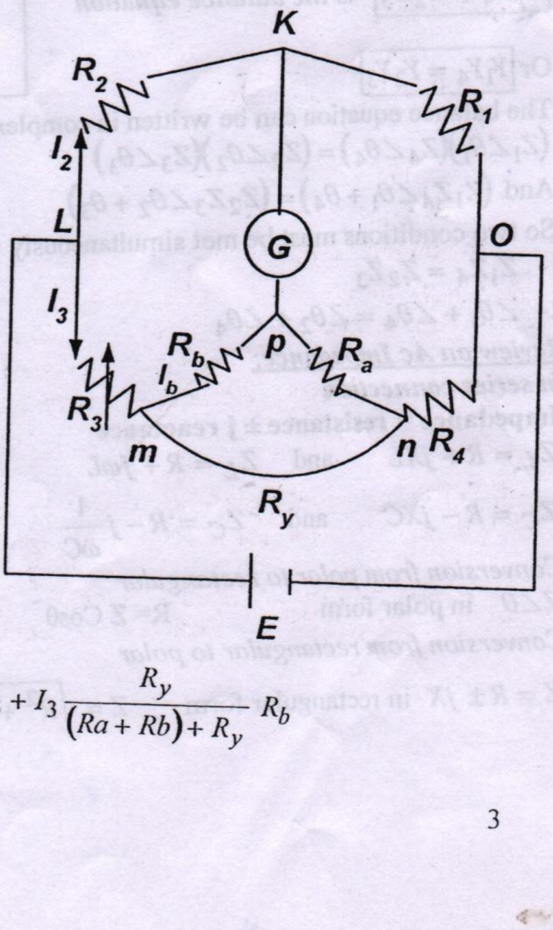
At balance $V_2 = V_3 + V_b \ldots\ldots\ldots (1)$

$$V_2 = E\frac{R_2}{R_1 + R_2} \quad \ldots\ldots (2)$$

$$V_3 = I_3 R_3 \quad \text{and} \quad V_b = I_b R_b \quad \ldots.. (3)$$

$$I_b = I_3 \frac{R_y}{(Ra + Rb) + R_y} \ldots\ldots\ldots\ldots (4)$$

$$E = I_3\left[R_3 + \frac{(Ra + Rb)R_y}{(Ra + Rb) + R_y} + R_4\right] \ldots (5)$$



Sub.equ. (5) in to equ. (2) and equ. (4) into equ.(3) then substitute the result in equ.(1), we get

$$I_3\left[R_3 + \frac{(Ra + Rb)R_y}{(Ra + Rb) + R_y} + R_4\right]\frac{R_2}{R_1 + R_2} = I_3 R_3 + I_3 \frac{R_y}{(Ra + Rb) + R_y} R_b$$

$$R_x = \frac{R_3 R_1}{R_2} + \frac{R_y Rb}{Ra + Rb + R_y}\left[\frac{R_1}{R_2} + 1 - 1 - \frac{Ra}{Rb}\right]$$

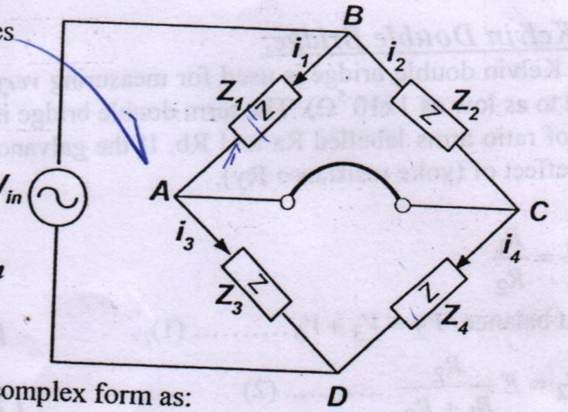
$$\boxed{R_x = \frac{R_3 R_1}{R_2} + \frac{R_y Rb}{Ra + Rb + R_y}\left[\frac{R_1}{R_2} - \frac{Ra}{Rb}\right]}$$ This is the balanced equation

If $\frac{R_a}{R_b} = \frac{R_1}{R_2}$ then $\boxed{R_x = \frac{R_3 R_1}{R_2}}$

2-*Ac Bridge and Their Application:*

The ac bridge is a natural outgrowth of the dc bridge and in its basic form consists of four bridge arms, a source of excitation, and a null ac detector. For measurements at low frequencies, the power line may serve as the source of excitation; but at higher frequencies an oscillator generally supplies the excitation voltage. The null ac detector in its cheapest effective form consists of a pair of headphones or may be oscilloscope.

The balance condition is reached when the detector response is zero or indicates null. Then $V_{AC} = 0$ and $V_{Z1} = V_{Z2}$



$$V_{Z1} = Vin\frac{Z_1}{Z_1 + Z_3}$$

$$V_{Z2} = Vin\frac{Z_2}{Z_2 + Z_4}$$ thus

$\boxed{Z_1 Z_4 = Z_2 Z_3}$ ***is the balance equation***

Or $\boxed{Y_1 Y_4 = Y_2 Y_3}$

The balance equation can be written in complex form as:

$$(Z_1\angle\theta_1)(Z_4\angle\theta_4) = (Z_2\angle\theta_2)(Z_3\angle\theta_3)$$

And $(Z_1 Z_4\angle\theta_1 + \theta_4) = (Z_2 Z_3\angle\theta_2 + \theta_3)$

So two conditions must be met simultaneously when balancing an ac bridge

1- $Z_1 Z_4 = Z_2 Z_3$

2- $\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_4$

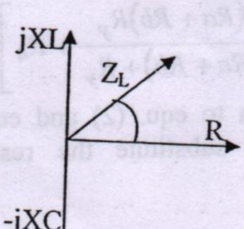
Review on Ac Impedance:

a) ***In series connection***

Impedance = resistance ± j reactance

$Z_L = R + jXL$ and $Z_L = R + j\omega L$

$Z_C = R - jXC$ and $Z_C = R - j\frac{1}{\omega C}$



Conversion from polar to rectangular

$Z\angle\theta$ in polar form R= Z Cosθ X= Z Sinθ become $Z = R \pm jX$

Conversion from rectangular to polar

$Z = R \pm jX$ in rectangular form $Z = \sqrt{R^2 + X^2}$ $\theta = \tan^{-1}\frac{X}{R}$ $\tan\theta = \frac{X}{R}$

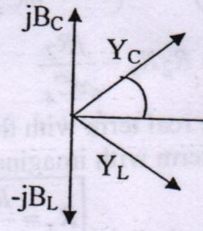
b) ***In parallel connection***

Admittance = conductance ± j susceptance

$$Y_L = G - jB_L \quad \text{and} \quad Y_L = \frac{1}{R} - j\frac{1}{\omega L}$$

$$Y_C = G + jB_C \quad \text{and} \quad Y_C = \frac{1}{R} + j\omega C$$

$$\tan\theta = \frac{B_C}{G} = \frac{\frac{1}{Xc}}{\frac{1}{R}} = \frac{\omega C}{\frac{1}{R}} = \omega RC$$



Example (1):

The impedance of the basic a.c bridge are given as follows:

$Z_1 = 100\angle 80°$ (inductive impedance) $\quad Z_2 = 250\Omega \quad Z_3 = 400\angle 30°$ (inductive impedance)

$Z_4 = unknown$

Sol:

$$\boxed{Z_4 = \frac{Z_2 Z_3}{Z_1}} \qquad Z_4 = \frac{250 \times 400}{100} = 1k\Omega \qquad \boxed{\theta_4 = \theta_2 + \theta_3 - \theta_1} \qquad \theta_4 = 0 + 30 - 80 = -50°$$

$Z_4 = 1000\angle -50°$ (capacitive impedance)

Example (2):

For the following bridge find Zx?

The balance equation $\quad Z_1 Z_4 = Z_2 Z_3$

$Z_1 = R = 450\Omega$

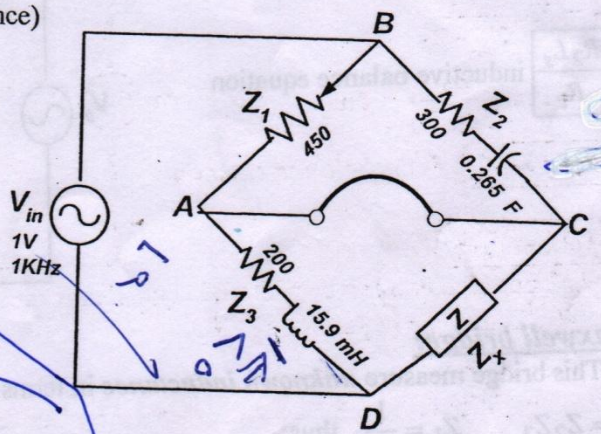
$Z_2 = R + \frac{1}{j\omega C} = R - \frac{j}{\omega C}$

$Z_2 = 300 - j600$

$Z_3 = R + j\omega L$

$Z_3 = 200 + j100$

$Z_4 = Z_x = unknown$



$$Z_4 = \frac{Z_2 Z_3}{Z_1} \qquad Z_4 = \frac{(300 - j600)(200 + j100)}{450} = 266.6 - j200$$

$$R = 266.6\Omega \qquad C = \frac{1}{2\pi F \times 200} = 0.79\mu F$$

a) Comparison Bridges:

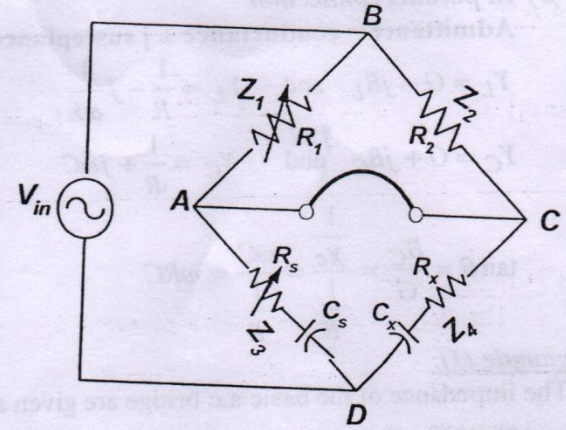
A.c comparison bridges are used to measure unknown inductance or capacitance by comparing it with a known inductance or capacitance.

1- ***Capacitive Comparison Bridge***:

In capacitive comparison bridge R1 & R2 are ratio arms, Rs in series with Cs are standard known arm, and Cx represent unknown capacitance with its leakage resistance Rx.

$$Z_1 = R_1 \qquad Z_2 = R_2 \qquad Z_3 = R_s - \frac{j}{\omega C_s} \qquad Z_4 = R_x - \frac{j}{\omega C_x}$$

At balance $\boxed{Z_1 Z_4 = Z_2 Z_3}$



$$R_1\left(R_x-\frac{j}{\omega C_x}\right)=R_2\left(R_s-\frac{j}{\omega C_s}\right)$$

$$R_1R_x-\frac{jR_1}{\omega C_x}=R_2R_s-\frac{jR_2}{\omega C_s}$$

By equating the real term with the real and imaginary term with imaginary we get:

$$R_1R_x=R_2R_s \qquad \boxed{R_x=\frac{R_2R_s}{R_1}}$$

$$\frac{-jR_1}{\omega C_x}=\frac{-jR_2}{\omega C_s} \qquad \boxed{C_x=\frac{R_1C_s}{R_2}}$$

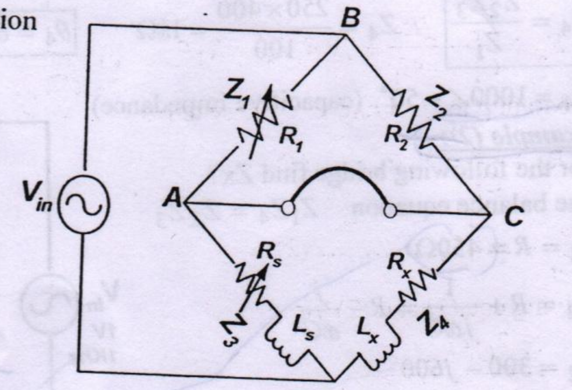
We can ***note*** that the bridge is ***independent*** on ***frequency*** of applied source.

2-*Inductive Comparison Bridge:*

The unknown inductance is determined by comparing it with a known standard inductor.

At balance we get

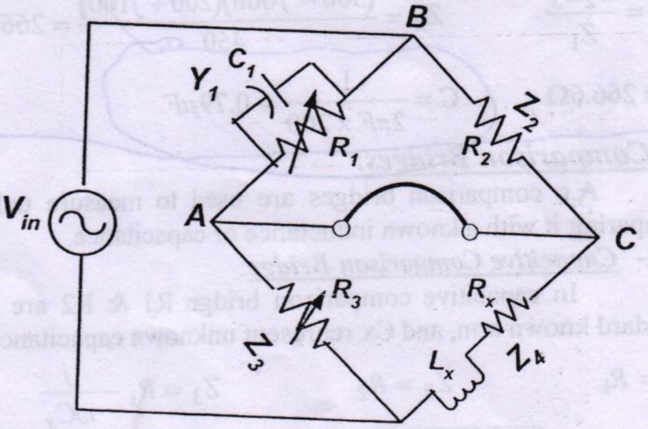
$\boxed{R_x=\frac{R_2R_s}{R_1}}$ represent resistive balance equation



$\boxed{L_x=\frac{R_2L_s}{R_1}}$ inductive balance equation

b) *Maxwell bridge:*

This bridge measure ***unknown inductance*** in terms of ***a known capacitance***, at balance:



$Z_1Z_4=Z_2Z_3 \qquad Z_1=\frac{1}{Y_1}$ thus

$\boxed{Z_4=Z_2Z_3Y_1}$ where

$Z_2=R_2 \qquad Z_3=R_3 \quad Y_1=\frac{1}{R_1}+j\omega C_1$

$Z_4=R_x+j\omega L_x$

So

$$R_x+j\omega L_x=R_2R_3\left(\frac{1}{R_1}+j\omega C_1\right)$$

$$R_x=\frac{R_2R_3}{R_1}$$

$$L_x=R_2R_3C_1$$

Maxwell bridge is limited to the measurement of ***medium quality factor (Q) coil*** with range between $1<Q\leq 10$

$$\left|\tan\theta_1\right| = \left|\tan\theta_4\right| = \frac{\omega L_4}{R_4} = \frac{B_{c1}}{G_1} = \frac{\frac{1}{XC_1}}{\frac{1}{R_1}} = \omega R_1 C_1 = Q$$

c) Hay Bridge:

Hay bridge convening for ***measuring high Q coils***

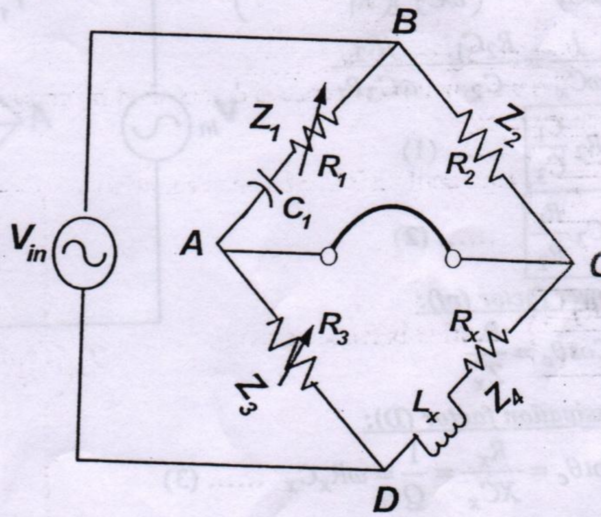
$$Z_1 = R_1 - \frac{j}{\omega C_1} \qquad Z_2 = R_2 \qquad Z_3 = R_3$$

$$Z_4 = R_x + j\omega L_x$$

At balance $\quad Z_1 Z_4 = Z_2 Z_3$

$$\left(R_1 - \frac{j}{\omega C_1}\right)\left(R_x + j\omega L_x\right) = R_2 R_3$$

$$R_1 R_x + \frac{L_x}{C_1} - \frac{jR_x}{\omega C_1} + j\omega R_1 L_x = R_2 R_3$$



Separating the real and imaginary terms

$$R_1 R_x + \frac{L_x}{C_1} = R_2 R_3 \quad ..... (1)$$

$$\frac{R_x}{\omega C_1} = \omega R_1 L_x \quad ...... \; ... (2)$$

Solving equ.(1) and (2) yields

$$R_x = \frac{\omega^2 C_1^2 R_1 R_2 R_3}{1+\omega^2 C_1^2 R_1^2}$$

$$L_x = \frac{R_2 R_3 C_1}{1+\omega^2 C_1^2 R_1^2}$$

$\theta 1 = -\theta 4$ because $\theta 2 = \theta 3 = zero$

$$\left|\tan\theta_1\right| = \left|\tan\theta_4\right| = \frac{\omega L_4}{R_4} = \frac{XC_1}{R_1} = \frac{\frac{1}{\omega C_1}}{R_1} = \frac{1}{\omega C_1 R_1} = Q$$

Thus $\quad Q = \frac{1}{\omega R_1 C_1} \quad .... (3)$

Submitted equ.(3) in to equ. (2) yield

$L_x = \frac{R_2 R_3 C_1}{1+\left(\frac{1}{Q}\right)^2}$ For Q> 10, then $\left(\frac{1}{Q}\right)^2 \ll 1$ and can be neglected, then $\boxed{L_x = R_2 R_3 C_1}$

d) ***Schering Bridge:***

Schering bridge used extensively for capacitive measurement, (C3) is standard high mica capacitor for general measurement work, or (C3) may be an air capacitor for insulation measurements. The balance condition require that $\theta1+\theta4=\theta2+\theta3$ but $\theta1+\theta4=-90$
Thus $\theta2+\theta3$ must equal (-90) to get balance

At balance $\boxed{Z_4 = Z_2 Z_3 Y_1}$

$$Y_1 = \frac{1}{R_1} + j\omega C_1 \qquad Z_2 = R_2 \qquad Z_3 = \frac{-j}{\omega C_3}$$

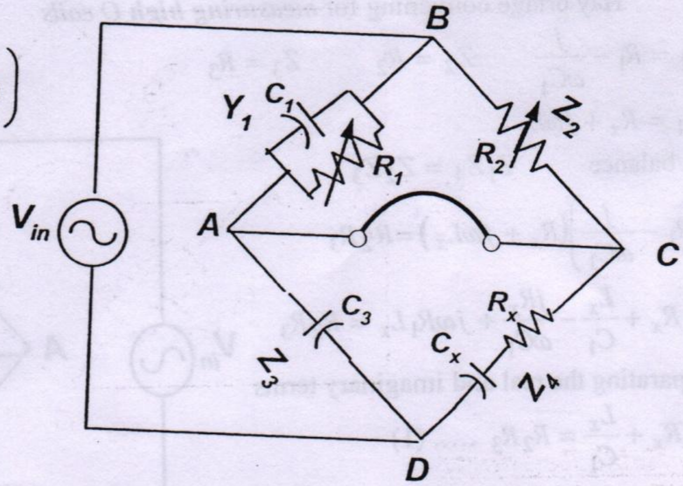
$$Z_4 = R_x - \frac{j}{\omega C_x}$$

$$R_x - \frac{j}{\omega C_x} = R_2\left(\frac{-j}{\omega C_3}\right)\left(\frac{1}{R_1} + j\omega C_1\right)$$

$$R_x - \frac{j}{\omega C_x} = \frac{R_2 C_1}{C_2} - \frac{jR_2}{\omega C_3 R_1}$$

$$\boxed{R_x = R_2 \frac{C_1}{C_3}} \quad \ldots\ldots (1)$$

$$\boxed{C_x = C_3 \frac{R_1}{R_2}} \quad \ldots\ldots (2)$$



The power factor (pf):

$$pf = Cos\theta_c = \frac{R_x}{Z_x}$$

The dissipation factor (D):

$$D = Cot\theta_c = \frac{R_x}{XC_x} = \frac{1}{Q} = \omega R_x C_x \quad \ldots\ldots (3)$$

Substitute equs. (1) & (2) into (3), we get

$$D = \omega R_1 C_1$$

e) ***Wien Bridge:***

This bridge is used to measured ***unknown frequency***

$$Z_1 = R_1 - \frac{j}{\omega C_1} \qquad Z_2 = R_2 \qquad Y_3 = \frac{1}{R_3} + j\omega C_3 \qquad Z_4 = R_4$$

$$Z_1 Z_4 = \frac{Z_2}{Y_3}$$

$$Z_2 = Z_1 Z_4 Y_3$$

$$R_2 = \left(R_1 - \frac{j}{\omega C_1}\right) R_4 \left(\frac{1}{R_3} + j\omega C_3\right)$$

$$R_2 = \frac{R_1 R_4}{R_3} + \frac{R_4 C_3}{C_1}$$

Dividing by R_4 we get

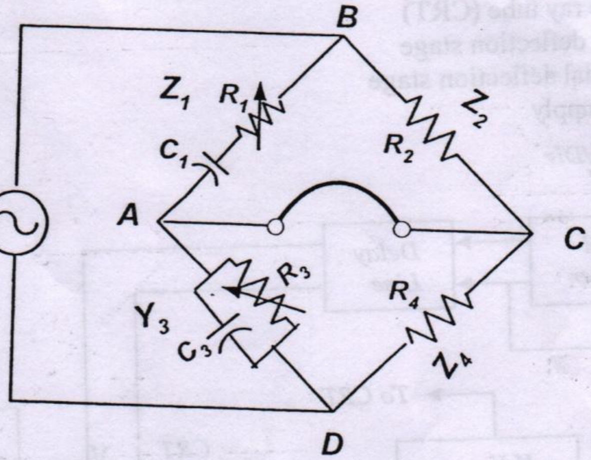
$$\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1} \quad \ldots\ldots (1)$$

Equating the imaginary terms, yield

$$\omega C_3 R_1 R_4 = \frac{R_4}{\omega C_1 R_3} \quad \text{Since } \omega = 2\pi F$$

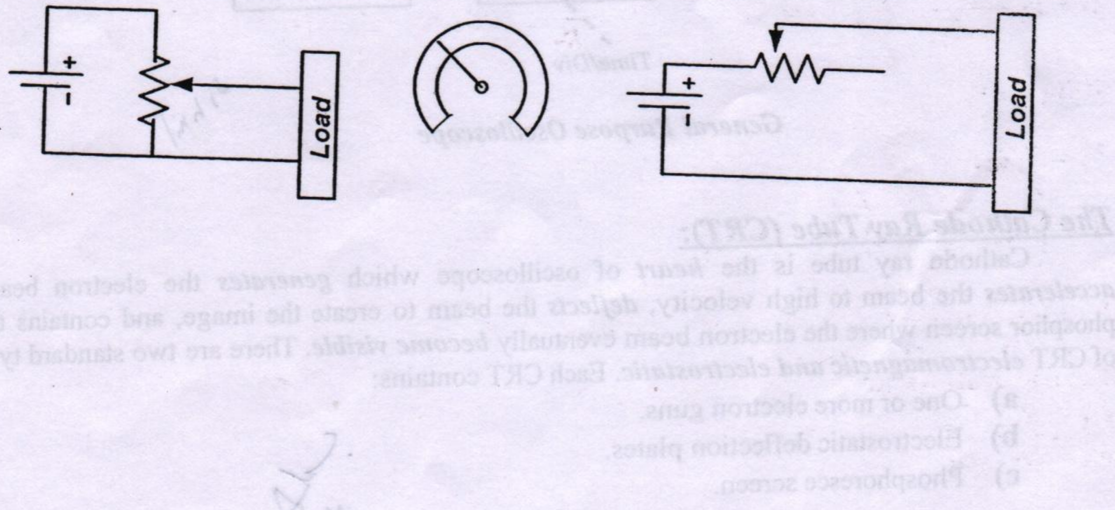
Thus $\boxed{F = \dfrac{1}{2\pi\sqrt{C_1 C_3 R_1 R_3}}}$ if $R_1 = R_3$ and $C_1 = C_3$ then $\dfrac{R_2}{R_4} = 2$ in equ.(1)

And $F = \dfrac{1}{2\pi RC}$ ***this is the general equation for Wien bridge***



Variable Resistors:

The variable resistance usually have three leads, two fixed and one movable. If the contacts are made to only ***two leads*** of the resistor (***stationary lead and moving lead***), the variable resistance is being employed as ***a rheostat*** which ***limit the current flowing*** in circuit branches. If all ***three contacts*** are used in a circuit, it is termed ***a potentiometer*** or pot and often used as ***voltage dividers*** to control or vary voltage across a circuit branch.



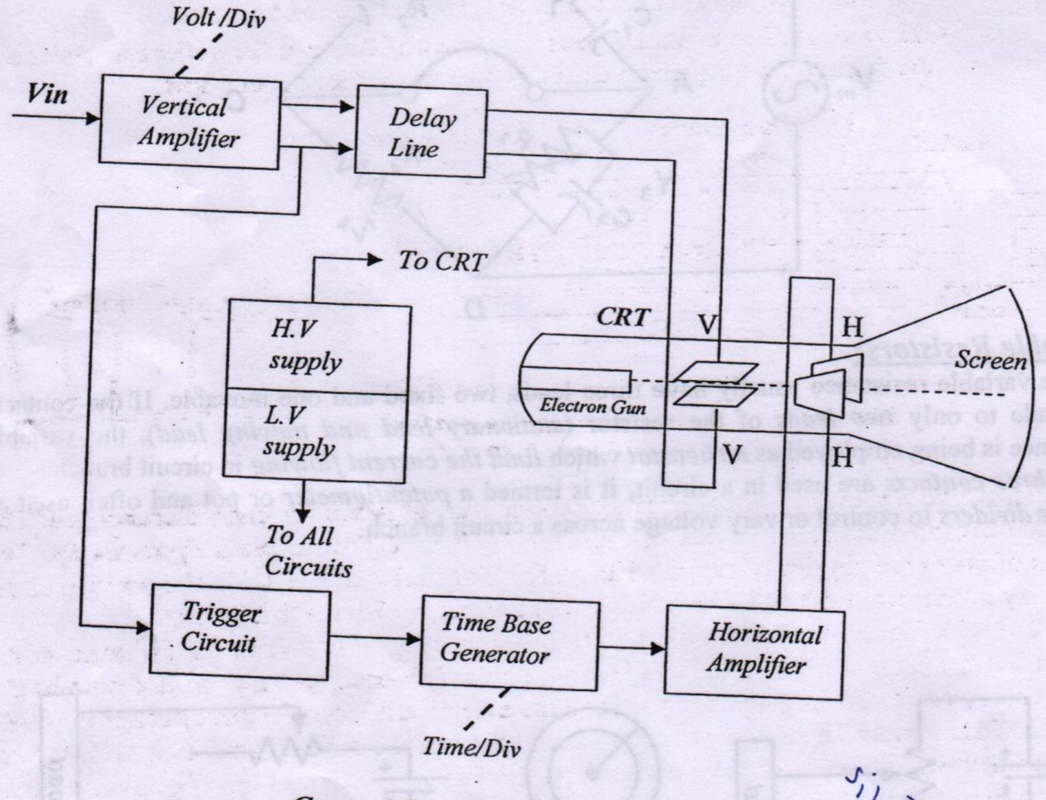
Oscilloscope

The ***cathode ray oscilloscope (CRO)*** is a device that allows the ***amplitude*** of electrical signals, whether they are voltage, current; power, etc., to be displayed primarily as a function of ***time***. The oscilloscope depends on the movement of an electron beam, which is then made visible by allowing the beam to impinge on a phosphor surface, which produces a visible spot

Oscilloscope Block Diagram:

General oscilloscope consists of the following parts:

1. Cathode ray tube (CRT)
2. Vertical deflection stage
3. Horizontal deflection stage
4. Power supply



General Purpose Oscilloscope

The Cathode Ray Tube (CRT):

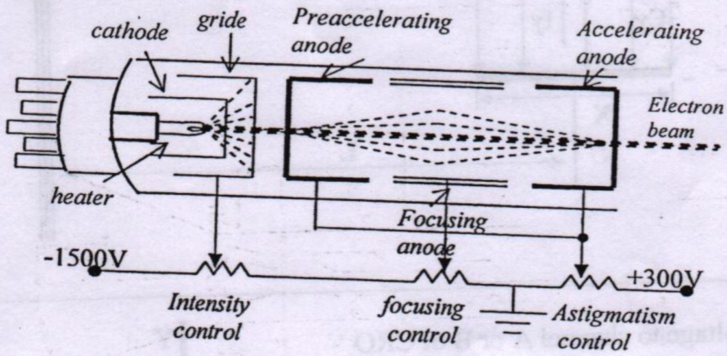
Cathode ray tube is the ***heart*** of oscilloscope which ***generates*** the electron beam, ***accelerates*** the beam to high velocity, ***deflects*** the beam to create the image, and contains the phosphor screen where the electron beam eventually ***become visible***. There are two standard type of CRT ***electromagnetic and electrostatic***. Each CRT contains:

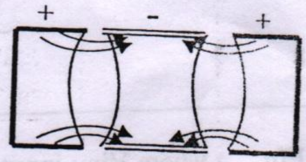
a) One or more electron guns.
b) Electrostatic deflection plates.
c) Phosphoresce screen.

A gun consists of a ***heated cathode, control grid, and three anodes.***

A ***heated cathod***e emits electrons, which are accelerated to the first accelerating anode, through a small hole in the ***control grid.*** The amount of cathode current, which governs the ***intensity*** of the spot, can be controlled with the control grid. The ***preaccelerating anode*** is a hollow cylinder that is at potential a few hundred volts more positive than the cathode so that the electron beam will be accelerated in the electric field. ***A focusing anode*** is mounted just a head of the preaccelerating anode and is also a cylinder. Following the focusing anode is the ***accelerating anode***, which gives the electron beam its last addition of energy before its journey to the deflecting plates. The focusing and accelerating anodes form an electrostatic lens, which bring the electron beam into spot focus on the screen. Three controls are associated with the operating voltages of the CRT; ***intensity, focus, and astigmatism***

1- The intensity control varies the potential between the cathode and the control grid and simply adjusts the beam current in the tube.
2- The focus control adjusts the focal length of the electrostatic lens.
3- The astigmatism control adjusts the potential between the deflection plates and the first accelerating electrode and is used to produce a round spot.





The ***electrostatic deflection*** system consists of ***two sets of plates*** for each electron gun. The ***vertical plates*** move the beam ***up and down,*** while ***horizontal plat***es move it ***right and left***. The two sets of plates are physically separated to prevent interaction of the field. The position of the spot at any instant is a resultant of potentials on the two set of plates at that instant.

The viewing ***screen*** is created by ***phosphor coating*** inside front of the tube. When electron beam strikes the screen of CRT with considerable energy, the phosphor absorbs the kinetic energy of bombarding electrons and reemits energy at a lower frequency range in visible spectrum. Thus a spot of light is produced in outside front of the screen. In addition to ***light, heat*** as well as ***secondary electrons*** of low energy is generating. ***Aquadag coating*** of graphite material is cover the inside surface of CRT nearly up the screen to remove these secondary electrons.

The property of some crystalline materials such as ***phosphor or zinc oxide*** *to emit light when stimulates by radiation is called* ***fluorescence.***

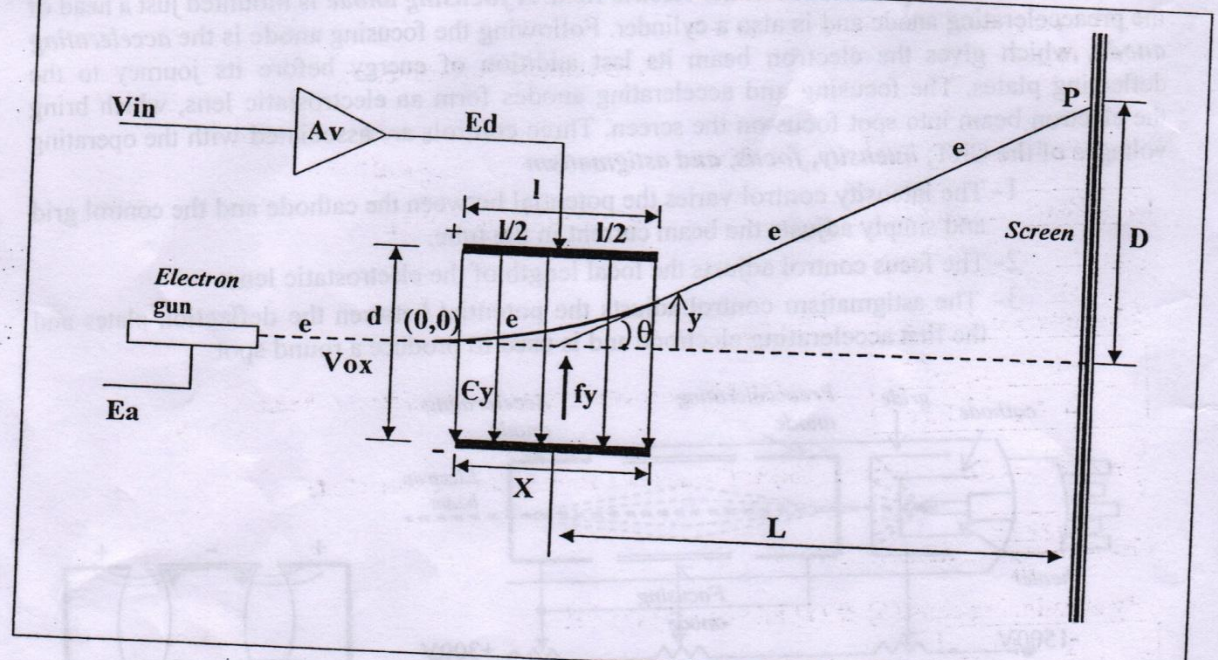
Phosphorescence refers to the property of material to continue ***light emission*** even after the source of excitation is ***cut off***.

Persistence is the length of ***time*** that the intensity of spot is taken to ***decrease*** to 10% of its original brightness.

Finally, the working parts of a CRT are enclosed in a high vacuum glass envelope to permit the electron beam moves freely from one end to other with out collision.

Graticules is a set of horizontal and vertical lines permanently scribed on CRT face to allow easily measured the waveform values.

Electrostatic Deflection Equations:



Vin where Vin : input voltage to channel A or B of CRO

$\Downarrow$

$Ed = Vin.Av$ **Ed: deflection voltage (potential)**

$\Downarrow$ $Ex = Ez = 0$

$Єy = \frac{-Ed}{d}$ …… (1) Єy: electrical field in Y direction

$\Downarrow$ Єx= Єz=0

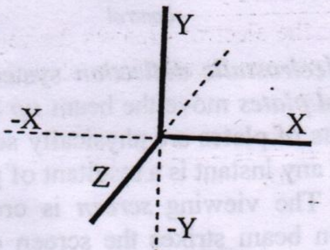
$fy = -e\ .Єy$ Fy: force generate by electrical field effect

$\Downarrow$ e: electron charge (1.6×10^{-19} C)

$ay = \frac{fy}{m_e}$ ay: acceleration in Y direction , $fx = fz = 0$

$ax = az = 0$

$\Downarrow$ m_e: electron mass (9.1×10^{-31} Kg)



$Vy = Voy + ayt$ Since $Voy = 0$ $Vx = Vox = constant$ $Vz = 0$

Vy: velocity in Y direction at any time

$Vy = ayt = \frac{fy}{m_e}t = \frac{-e}{m_e}Єy\ t$

Voy: initial velocity in Y direction

$\Downarrow$

$Y = Yo + Voy + \frac{1}{2}ayt^2$ Since $Yo = 0$ $Voy = 0$ Y: distance in Y direction

$$Y = \frac{1}{2}ayt^2 = \frac{-1}{2}\frac{e}{m_e}\ \epsilon y\, t^2$$ Yo: initial distance in Y direction

$$\boxed{Y = \frac{-1}{2}\frac{e}{m_e}\ \epsilon y\, t^2}\ \ldots\ldots\ldots\ldots (2)$$ Relation of Y with time

$Vx = Vox + axt$ Since $ax = 0$

$Vx = Vox$ Vx: velocity in X direction

⇓

$X = Xo + Voxt + \frac{1}{2}axt^2$ Since $Xo = 0$ $\frac{1}{2}axt^2 = 0$

$$\boxed{X = Voxt}\ \ldots\ldots\ldots (3)$$ Relation of X with time

$$t = \frac{X}{Vox}\ \ldots\ldots\ldots\ldots (4)$$

Substitute equ. (4) into equ.(2) give

$$\boxed{Y = \frac{-1}{2}\frac{e}{m_e}\ \epsilon y \frac{X^2}{Vox^2}}\ \ldots\ldots (5)$$ ***The parabolic equation of electron beam***

$\frac{1}{2}mVox^2 = eEa$ **where (Ea) is the acceleration voltage (potential)**

$$Vox = \sqrt{\frac{2eEa}{m}}\ \ldots\ldots\ldots\ldots\ldots (6)$$

By substituting equs.(6) & (1) into equ.(5) we get

$$\boxed{Y = \left(\frac{1}{4d}\frac{Ed}{Ea}\right).X^2}\ \ldots\ldots\ldots (7)$$ ⇐ ***Relation of Y with X***

When the electrons leaves the region of deflecting plates, the deflecting force no longer exist, and the electrons travels in a straight line toward point P. The slope of parabolic curve at distance (x=l) is:

$$\tan\theta = \frac{dy}{dx} = \frac{-el}{mVox^2}\ \epsilon y$$

Or

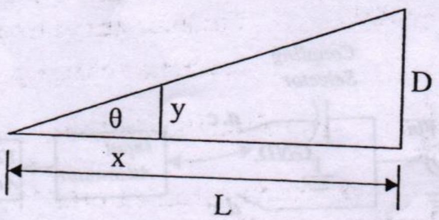
$$\tan\theta = \left(\frac{1}{2d}\frac{Ed}{Ea}\right)l\ \ldots\ldots\ldots (8)$$

The deflection on the screen (D) is

$$D = L\tan\theta\ \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Substitute equ.(9) into (8) give

$$\boxed{D = \frac{lLEd}{2dEa}}\ \ldots\ldots\ldots\ldots\ldots (10)$$



By similarity

$$\boxed{\frac{y}{x} = \frac{D}{L}}$$

♣ ***The deflection sensitivity (S) of CRT is:***

$$S = \frac{D}{Ed}\ \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

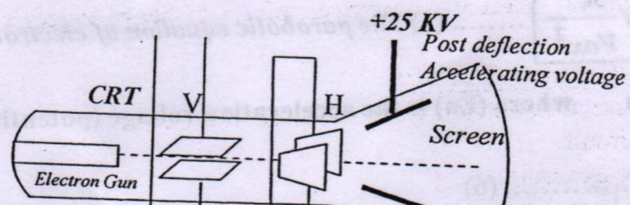
♣ ***The deflection factor (G) of CRT is:***

$$G = \frac{1}{S} = \frac{Ed}{D} = \frac{2dEa}{lL}\ \ldots\ldots (12)$$

Post Deflection Acceleration:

The amount of light given off by the phosphor depends on the amount of energy that is transferred to the phosphor by the electron beam. For ***fast oscilloscope*** (of high frequency response greater than 100MHz), the ***velocity*** of electron beam must be ***great*** to respond to fast occurring events; otherwise, the light output will be drop off. This is done by increasing the acceleration potential but it will be difficult to deflected the fast electron beam by the deflection plates because this would required a higher deflection voltage and a higher deflection current to charge the capacitance of the plates.

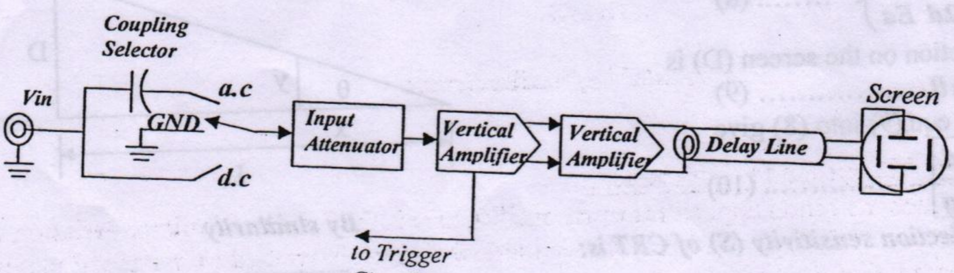
Modern CRTs use a two step acceleration to eliminate this problem. ***First,*** the electron beam is ***accelerated*** to a relatively ***low velocity*** through a potential of a few thousand volts. The beam is then deflected and ***after deflection is further accelerated to the desired final velocity.*** The ***deflection sensitivity*** of the CRT ***depends*** on the ***acceleration voltage*** before the deflection plates, which is usually regulated and does not depends on the post acceleration voltage after the deflection plates.



Vertical deflection system:

The vertical deflection system provides an amplified signal of the proper level to derive the vertical deflection plates with out introducing any appreciable distortion into the system. This system is consists of the following elements:

1- Input coupling selector.
2- Input attenuator.
3- Preamplifier.
4- Main vertical amplifier.
5- Delay line.



Vertical Deflection System

1- ***Input Coupling Selector:***

Its purpose is to allow the oscilloscope more flexibility in the display of certain types of signals. For example, an input signal may be a d.c signal, an a.c signal, or a.c component superimposed on a d.c component. There are three positions switch in the coupling selector (d.c, a.c, and GND). If an ***a.c*** position is chosen, the capacitor appears as an open circuit to the d.c components and hence block them from entering. While the GND position ground the internal circuitry of the amplifier to remove any stored charge and recenter the electron beam.

2- ***4- Input Attenuators And Amplifiers:***

The combine operation of the attenuator, preamplifier and main amplifier together make up the amplifying portion of the system.

The function of the attenuator is to reduce the amplitude of the input signal by a selected factor and verse varies amplifier function.

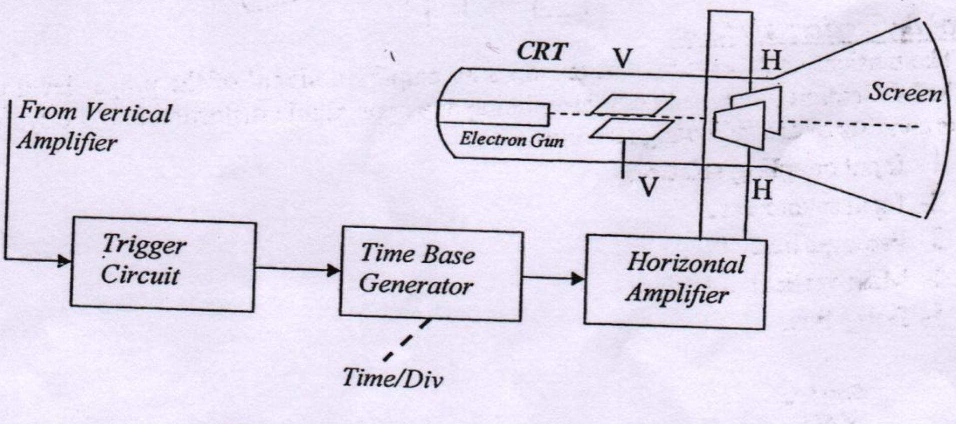
5- ***Delay Line:***

Since part of the input signal is picked off and fed to the horizontal deflection system to initiate a sweep waveform that is synchronized with the leading edge of the input signal. So the purpose of delay is to delay the vertical amplified signal from reaching the vertical plates until the horizontal signal reach the horizontal plates to begin together at the same time on CRT screen.

Horizontal Deflection System:

The horizontal deflection system of OSC consist of :

1- Trigger circuit.
2- Time base generator.
3- Horizontal amplifier.



Horizontal Deflection System

Trigger and Time Base Generator:

The most common application of an oscilloscope is the display of voltage variation versus time. To generate this type of display a saw tooth waveform is applied to horizontal plates. The electron beam being bent towards the more positive plate and deflected the luminous spot from left to right of the screen at constant velocity whilst the return or fly back is at a speed in excess of the maximum writing speed and hence invisible. The saw tooth or time base signal must be repetitively applied to the horizontal plates so that; the beam can retrace the same path rapidly enough to make the moving spot of light appear to be a solid line.

To synchronous the time base signal applied to (X-plates) with input voltage to be measured which applied to vertical or (Y-plates) a triggering circuit is used. This circuit is sensitive to the level of voltage applied to it, so that when a predetermined level of voltage is reached a pulse is passed from the trigger circuit to initiate one sweep of the time base. In a practical oscilloscope the time base will be adjustable from the front panel control of scope.

Horizontal Amplifier:

The horizontal amplifier is used to amplify the sweep waveform to the required level of horizontal plates operation.

Transducers

The input quantity for most instrumentation system is nonelectrical. In order to use electrical methods and techniques for measurement, manipulation, or control the nonelectrical quantity is converted into an electrical signal by a device called ***electrical transducer***.

Transducers are broadly defined as devices that convert energy or information from one form to another. This energy may be electrical, mechanical, chemical, optical (radiant), or thermal. Such as, for example, mechanical force or displacement, liner and angular velocity heat, light intensity, humidity, temperature variation, sound time, pressure, all are converted into electrical energy by means of electrical transducers.

Transducers may be classified according to their application, method of energy conversion, nature of output signal, and so on.

1- *Primary and Secondary Transducers:*

Transducers, on the basis of ***methods of applications***, may be classified into *primary* and *secondary* transducers. Transducer that converts ***energy from any form to electrical form*** is called ***primary*** transducer, such as a *photovoltaic cell*, while transducer that converts energy from ***any form to another form but not electrical energy or signal*** is called ***secondary*** transducer, such as *displacement transducer* (which converts force or pressure to displacement).

2- *Active and Passive Transducers:*

Transducers, on the basis of ***methods of energy conversion*** used, may be classified into *active* and *passive* transducers.

Active (self generating) transducers develop their output in the form of electrical voltage or current ***without any source of electrical excitation***, such as ***thermocouples, tacho generator***.

While, if the transducer is capable of producing an output signal only when it is ***in connection with electrical power source*** is called ***passive*** transducers, such as ***a potentiometer, thermistor*** (thermal resistance). Passive transducer producing a variation in some electrical parameter, such as a resistance, capacitance, inductance and so on, which can be measured as voltage or current variation in the circuit.

3- *Analog and Digital Transducers:*

Transducers, on the basis of ***nature of output signal***, may be classified into *analog* and *digital* transducer.

Analog transducer converts input signal into output signal, which is ***a continuous function*** of time, while ***digital*** transducer converts input signal into output signal in ***a discrete forms***.

Strain Gauges

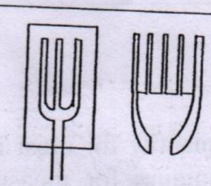
The strain gauge is an example of a primary passive analog transducer that converts force or small displacement into a change of resistance. Since many other quantities such as torque, pressure, weight, and tension also involve force or displacement effects, they can also be measured by strain gauges.

Strain gauges are so named because when they undergo a strain (defined to be a fractional change in linear dimension tension or compression caused by an applied force); they also undergo a change in electrical resistance. The strain takes the form of a lengthening of the special wire from which the gauge is constructed. The change in resistance is proportional to the applied strain and is measured with a specially adopted Wheatstone bridge.

The gauge factor (k) is given by:

$$\boxed{k = \frac{\Delta R/R}{\Delta l/l}}$$, where $R = \delta \frac{l}{A}$ and $R = \delta \frac{l}{(\Pi/4)d^2}$ since the strain $(\sigma) = \frac{\Delta l}{l}$ thus

$$k = \frac{\Delta R/R}{\sigma} \quad \text{and} \quad \Delta R = kR\sigma$$



Displacement transducers

The mechanical elements that are used to ***convert*** the applied ***force into a displacement*** are called ***force summing devices***. The force summing members generally used the following:

a) Diaphragm, flat or corrugated b) Bellows c) Bourdon tube, circular or twisted
d) Straight tube e) Mass cantilever, single or double suspension f) Pivot torque

The displacement created by the action of the force summing device is converted into a change of some electrical parameter and measured by one of the following electrical principle:

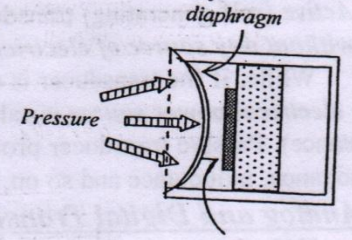
1) Capacitive 2) Inductive 3) Differential transformer 4) Photoelectrical
5) Potentiometer 6) Ionization 7) Oscillation 8) Piezoelectric 9) Velocity

1- *Capacitive Transducer:*

The capacitance of a parallel plate capacitor is given by $C = \frac{\varepsilon_o \varepsilon_r A}{d}$, where $\varepsilon_o = 8.85 \times 10^{-12}$ F/m and ε_r = relative dielectric constant

A force applied to a diaphragm that function as one plate of a simple capacitor, change the distance between the diaphragm and static plate.

The resulting change in capacitance could be measured with an ac bridge, but it is usually measured with an oscillator circuit. The transducer as a part of oscillator circuit will causes a change in oscillator frequency which proportional to the applied force.

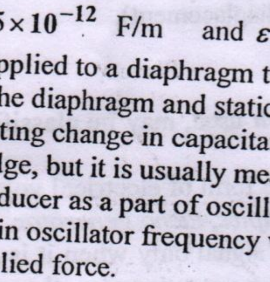


2- *Inductive Transducer:*

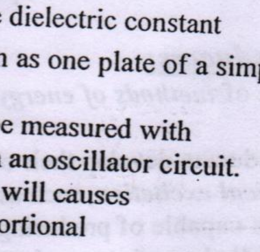
In the inductive transducer the measurement of force is accomplished by the change in the inductance ratio of a pair of coils or by the change of inductance in a single coil. The ferromagnetic armature is displaced by the force being measured, varying the reluctance of the magnetic circuit. The air gap is varied by a change in position of the armature; the resulting change in inductance is a measure of applied force.

force→displacement→air gap change→permaebility(μ) $\rightarrow \Re = l/\mu A \rightarrow$

$$\rightarrow \Phi = mmf/\Re = NI/\Re \rightarrow \; L = N\frac{\partial \Phi}{\partial i}$$



Double coil



Single coil

3- *Linear Variable Differential Transformer:*

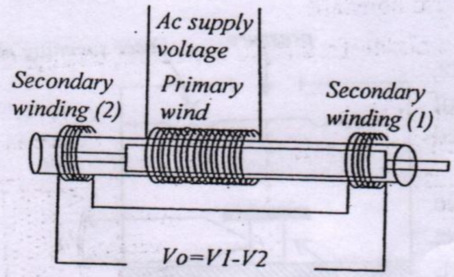
It is produces an electrical signal that is linearly proportional to mechanical displacement. The displacements detectable by LVDTs are relatively large compared to those detectable by strain gauges.

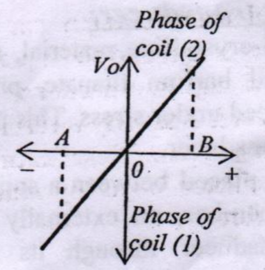
LVDT consists of a single primary winding and two secondary windings which are placed on either side of the primary. The secondary windings have an equal number of turns but they are connected in series opposition so that the emf induced in the coil oppose each other. The position of the moveable core determines the flux linkage between the ac excited primary winding and each of the two secondary winding.

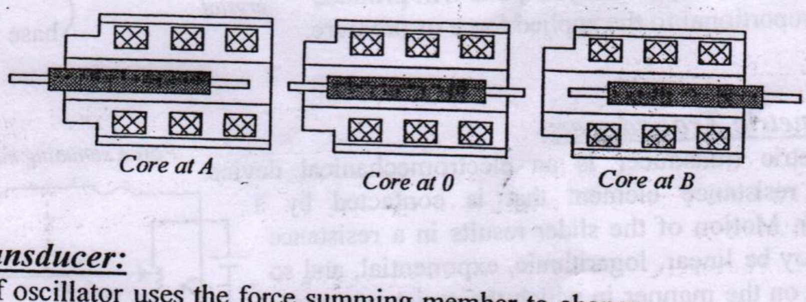
a) With the core in the centre, or reference position, the induced emfs in the secondaries are equal, and since they oppose each other, the output voltage will be zero.

b) When the core is forced to move to the left, more flux links the left hand coil than the right hand coil, the induced emf of the left coil is therefore larger than the induced emf of the right coil. The magnitude of the output voltage is than equal to the difference between the two secondary voltages, and it is in phase with the voltage of the left hand coil.

c) Similarly, when the core is force to move to the right, more flux links the right hand coil than the left hand coil and the result output is now in phase with the emf of the right hand coil, while its magnitude again equals the difference between the two induced emfs.





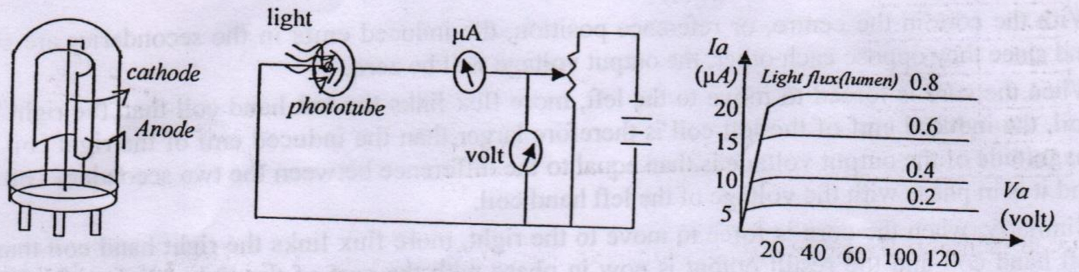


4- *Oscillator Transducer:*

This class of oscillator uses the force summing member to change the capacitance or inductance in an (LC) oscillator circuit, which change the frequency of the circuit in proportional to the applied force.

5- Photoelectric Transducer:

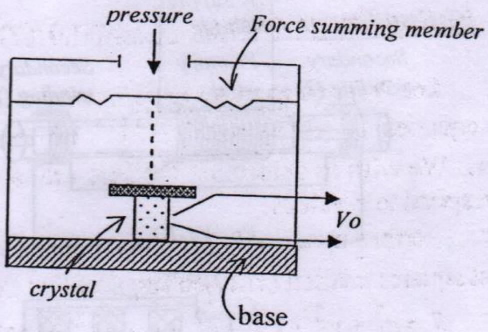
The photoelectric transducer makes use of the properties of photo emissive cell or phototube. The phototube is a radiant energy device that controls its electron emission when exposed to incident light. The large semicircular element is the photosensitive cathode and the thin wire down the centre of the tube is the anode. Both elements are place in a high vacuum glass envelope. When a constant voltage is applied between cathode and anode, the current is directly proportional to the amount of light falling on the cathode. The output current is extremely small in (µA) and for a voltage approximately above 20V, the output current is nearly independent of the applied anode voltage but depends entirely on the amount of the incident light.



6- Piezoelectric transducer:

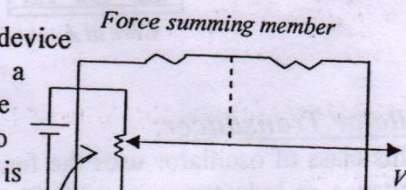
A symmetrical crystalline material, such as quartz, Rochelle salt, and barium titanate, produce an emf when they are placed under stress. This property is used in piezoelectric transducer.

When a crystal is placed between a solid base and the force summing member, an externally applied force, entering the transducer through its pressure port, applies pressure to the top of a crystal; this will produce a small emf proportional to the applied force or pressure.



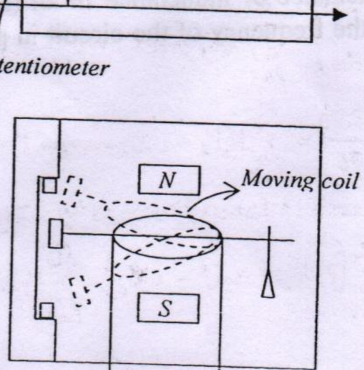
7- Potentiometric Transducer:

A Potentiometric transducer is an electromechanical device containing a resistance element that is contacted by a movable slider. Motion of the slider results in a resistance change that may be linear, logarithmic, exponential, and so on, depending on the manner in which the resistance wire is wound.



8- Velocity Transducer (Tachometer):

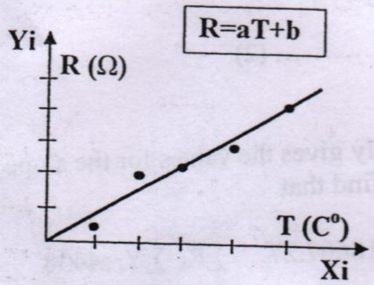
The velocity transducer essentially consists of a moving coil suspended in the magnetic field of a permanent magnet. A voltage is generated by the motion of the coil in the field which is proportional to the velocity of the moving coil. It is used for velocity measurement.

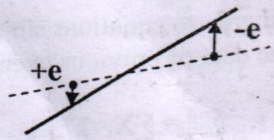


Curve Fitting and Approximation by Least Square Approximations Method

Suppose we wish to fit a curve to an approximate set of data such as from the determination of the effects of temperature on resistance, we want to suitably ***determine the constants*** **(a)** and **(b)** in the equation relating resistance **R** and temperature **T**, we shall assume that the error of reading the temperature is negligible, so that all the errors are in the resistance measurements, and use vertical distance.

T(C°)	R(Ω)
20.5	765
32.7	826
51.0	873
73.2	942
95.7	1032





We want to minimize the deviations of the points from the line (errors). The deviations are measured by *the distances from the point to the line*. The deviation could be minimizing by making their sum minimum, but this is not an adequate criterion.

1- *Fitting linear curves by Least Square Approximations (LSA) method:*

Let (Yi) represent an experimental value, and let (y_i) represent the theoretical value which get by this equation $y_i = aX_i + b$ where (X_i) is a particular value of the variable assumed free from error. We wish to determine the best values for **(a)** and **(b)**, so that (y_i) predict the function values that correspond to x-values.

Since error= measured- theoretical thus $e_i = Y_i - y_i$

Least squares method criterion requires that the sum of errors in the reading (S) must be minimized, so

$$S = e_1^2 + e_2^2 + e_3^2 + e_4^2 + \dots + e_N^2 = \min$$ where N is the number of the measured points

$$S = \sum_{i=1}^{N} e_i^2 = \sum_{i=1}^{N}(Y_i - y_i)^2 = \sum_{i=1}^{N}(Y_i - aX_i - b)^2 = \min$$ which reach the minimum by proper choice of parameters (a) and (b). When S be a minimum, the two partial derivatives $\left(\frac{\partial S}{\partial a}\right)$ and $\left(\frac{\partial S}{\partial b}\right)$ will be zero. Thus

$$\left(\frac{\partial S}{\partial a}\right) = \sum_{i=1}^{N} 2(Y_i - aX_i - b)(-X_i) = 0 \quad \dots\dots\dots\dots (1)$$

$$\left(\frac{\partial S}{\partial b}\right) = \sum_{i=1}^{N} 2(Y_i - aX_i - b)(-1) = 0 \quad \dots\dots\dots\dots (2)$$

Dividing each of these equations by (-2) and expanding the summation, we get

$$\sum_{i=1}^{N}\left(Y_i X_i - aX_i^2 - bX_i\right) = \sum_{i=1}^{N} Y_i X_i - a\sum_{i=1}^{N} X_i^2 - b\sum_{i=1}^{N} X_i = 0$$

$$\sum_{i=1}^{N}(Y_i - aX_i - b) = \sum_{i=1}^{N} Y_i - a\sum_{i=1}^{N} X_i - \sum_{i=1}^{N} b = 0 \qquad \textit{note that: } \boxed{\sum_{i=1}^{N} b = Nb}$$

The ***normalized equations***

$$\boxed{a\sum_{i=1}^{N} X_i^2 + b\sum_{i=1}^{N} X_i = \sum_{i=1}^{N} Y_i X_i} \text{ .... (1)}$$

$$\boxed{a\sum_{i=1}^{N} X_i + bN = \sum_{i=1}^{N} Y_i} \text{ .... (2)}$$

Solving these equations simultaneously gives the values for the slope and intercept (a) and (b). For the data of previous example, we find that

N=5 $\sum T_i = \sum X_i = 273.1$ $\sum T_i^2 = 18607.27$ $\sum R_i = \sum Y_i = 4438$ $\sum T_i R_i = \sum Y_i X_i = 2549325$

The normalized equations are then

18607.27 a + 273.1 b = 254932.5 .... (1)

273.1 a + 5 b = 4438 .... (2)

By solving these equations we obtain *a=3.395* , *b=702.17*

And hence the equation R=aT+b will be $\boxed{R = 3.39T + 702.17}$

To find R at T=70 is R(70)=3.39x70+702.17=939.3 Ω

The measured and theoretical values are then

$T(C^o)$	20.5	32.7	51.0	73.2	95.7
$R_{measured}$	765	826	873	942	1032
$R_{theoretical}$	772	813	875	950	1026

Exercise:

Find the theoretical values for linear equation (y) from the given data

Xi	6	8	10	12	14	16	18	20	22	24
Yi	3.8	3.7	4	3.9	4.3	4.2	4.2	4.4	4.5	4.5
yi										

a=0.0445 b=3.48

2- *Fitting nonlinear curves by Least Square Approximations method:*

In many cases, of course, data from experimental test are not linear, so we need to fit other function than a first degree polynomial, popular forms that are tried are the exponential forms such as

$\boxed{y = ax^b}$ or $\boxed{y = ae^{bX}}$

The exponential forms are usually linearized by taking logarithms before determining the parameters, so for the first equ. $\ln y = \ln a + b \ln X$, then let ln y=z , ln a= A, ln X=w

Thus $\boxed{z = A + bw}$ is a linear equation