Al-Rasheed University College
Medical Instrumentation Tech. Eng.

Measurements & medical Transducers

2nd Stage
Lecturer: Dr. Najim Abdallah

Lecture Two
Error of Measurement

2.1 Types of Errors

- In general, an error in measurement is defined as the difference between the measured value and the expected value.

- No measurement can be done perfectly. Errors are always existing. Therefore, it is important to know how different errors have entered into our measurements.

- Errors may come from different source and are usually classified under four main types:

1- Gross (Human) errors.

i) Misreading of instruments and observation errors.
ii) Improper choice of instrument, or the range of instrument.
iii) Incorrect adjustment or forgetting to zero.
iv) Erroneous calculations, computation mistakes, and estimation errors.
v) Neglect of loading effects.
vi) Proper position for measuring human.

2- Instrumentation (Equipment) Errors

i) Damaged equipment such as defective due to loading effect or worn parts.
ii) Calibration errors.
iii) Component nonlinearities.
iv) Loss during transmission.
v) Proper position of equipment (vertical or horizontal).
vi) Static charge error.

3- Environmental Errors

i) Change in temperature, pressure.
ii) Humidity.
iii) Stray electric and magnetic fields.
iv) Mechanical vibration.
v) Weather variations (day, night, and four seasons).

4- Random Errors

- These errors are due to unknown reasons and happen when all other errors are considered. Generally, random errors cannot be fully understood.

- The only ways to reduce this type of errors is by:

  - increasing the number of readings of the measured quantity.

  - Then, we apply mathematical and statistical analysis to determine the best estimate of the measured value.

2.2 Mathematical Equations of Errors

i- Error:

$$error = |Y - Xi| \tag{1}$$

where:

- Y : the expected value.
- Xi : the ith measured value.

Ii- Error rate:

$$\text{error rate} = \frac{|Y - X_i|}{Y} * 100\% \tag{2}$$

$$= \frac{error}{Y} * 100\% \tag{3}$$

iii- Accuracy:

$$\text{accuracy} = 100\% - \text{error rate} \tag{4}$$

Vi- Average:

$$\bar{X}_n = \frac{X_1 + X_2 + \cdots + X_n}{n} \tag{5}$$

V- Precision:

$$\text{precision} = 1 - \frac{|X_i - \bar{X}_n|}{\bar{X}_n} \tag{6}$$

where:

- Xi : the ith measurement (reading).
- $\overline{X_n}$: the average of n readings.

Example 1

Suppose we used an Ohmmeter to measure the value of a resistor. We got the following reading (20.33Ω, 20.18Ω, 20.24Ω, and 20.19Ω). Assume that the expected value is (20Ω).

Sol:

- the error for each one of our readings can be calculated as follows:

$e_1 = |20.33 - 20| = 0.33\Omega$
$e_2 = |20.18 - 20| = 0.18\Omega$
$e_3 = |20.24 - 20| = 0.24\Omega$
$e_4 = |20.19 - 20| = 0.19\Omega$

- error rate for each measurement can be calculated as follows:

$$\text{error rate}_1 = \frac{0.33}{20} * 100\% = 1.65\%$$
$$\text{error rate}_2 = \frac{0.18}{20} * 100\% = 0.9\%$$
$$\text{error rate}_3 = \frac{0.24}{20} * 100\% = 1.2\%$$
$$\text{error rate}_4 = \frac{0.19}{20} * 100\% = 0.95\%$$

- Accuracy for each reading can also be calculated as follows:

$$
\begin{aligned}
\text{accuracy}_1 &= 100\% - \text{error rate}_1 \\
&= 100\% - 1.65\% = 98.35\%
\end{aligned}
$$

$$
\begin{aligned}
\text{accuracy}_2 &= 99.1\% \\
\text{accuracy}_3 &= 98.8\% \\
\text{accuracy}_4 &= 99.05\%
\end{aligned}
$$

Example 2: Ten measurements were conducted in the lab and data was recorded in the following table. Calculate the precision of the 4th measurement value.

Sequence	Measurement Value (volts)
1	98
2	102
3	101
4	97
5	100
6	103
7	98
8	106
9	107
10	99

Sol:

- Precision of the 4th measurement value can be calculated using the following:

$$\text{precision}_4 = 1 - \frac{|X_4 - \bar{X}_{10}|}{\bar{X}_{10}}$$

- Average value can be calculated as:

$$\begin{aligned}\bar{X}_{10} &= \frac{98 + 102 + \cdots + 99}{10} \\ &= 101.1\end{aligned}$$

$$\begin{aligned}\Rightarrow \text{precision}_4 &= 1 - \frac{|97 - 101.1|}{101.1} \\ &= 0.96\end{aligned}$$

Example 3:

The expected value of the voltage across a resistor is 80 V. However, the measurement gives a value of 79 V. Calculate the (i)absolute error, (ii) % error, (iii) relative accuracy, and (iv) % of accuracy.

Ans.

(i) Absolute error $e = |Y_n - X_n| = 80 - 79 = 1\ \text{V}$

(ii) % Error $= \dfrac{|Y_n - X_n|}{Y_n} \times 100 = \dfrac{|80 - 79|}{80} \times 100 = 1.25\%$

(iii) Relative Accuracy

$$A = 1 - \left|\frac{Y_n - X_n}{Y_n}\right| = 1 - \left|\frac{80 - 79}{80}\right|$$

$\therefore \quad A = 1 - 1/80 = 79/80 = 0.9875$

(iv) % of Accuracy $\quad a = 100 \times A = 100 \times 0.9875 = 98.75\%$

or $\quad a = 100\% - \%\ \text{of error} = 100\% - 1.25\% = 98.75\%$

2.3 Random Errors Statistical analysis

tools to determine the best approximation of the measured data and avoiding the random errors in the measurements taken.

1. Arithmetic mean:

$$\bar{x} = \frac{x_1 + x_1 + x_1 + \ldots + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$\bar{x} \rightarrow$ Arithmatic mean

$x_i \rightarrow i^{th}$ measurement taken

n $\rightarrow$ Number of measurment taken

2. Deviation from the mean

- each reading of the measurement deviates from the mean by a certain amount. The closer to the mean, is the better approximation to the true value.

$$d_i = x_i - \overline{x}$$

$d_i \rightarrow$ deviation of i^{th} measurement taken from the mean.

$\overline{x} \rightarrow$ Arithmatic mean

$x_i \rightarrow i^{th}$ measurement taken

- The deviation, d can be either positive or negative. Note that the algebraic sum (NOT the absolute) of all the deviations equals zero.

2. Average deviation:

- the average deviation indicates the precision of the measuring instruments.

$$D = \frac{|d_1| + |d_1| + \ldots + |d_n|}{n} = \frac{\sum |d|}{n}$$

$d_i \rightarrow$ deviation of i^{th} measurement taken from the mean.

$\bar{x} \rightarrow$ Arithmatic mean

$x_i \rightarrow i^{th}$ measurement taken

Example 4:

A set of independent current measurements was taken by six observers and recorded as (12.8 mA, 12.2 mA, 12.5 mA, 13.1 mA, 12.9 mA, and 12.4 mA). Calculate (a) the arithmetic mean, (b) the deviation from the mean, (c) The average deviation from the mean.

Ans.

- The arithmetic mean equals:

$$\bar{x} = \frac{12.8 + 12.2 + 12.5 + 13.1 + 12.9 + 12.4}{6} = 12.65\text{mA}$$

- The deviations for each measurement from the mean are:

$d_1 = 12.8 - 12.65 = 0.15$ mA

$d_2 = 12.2 - 12.65 = 0.45$ mA

$d_3 = 12.5 - 12.65 = 0.15$ mA

$d_4 = 13.1 - 12.65 = 0.45$ mA

$d_5 = 12.9 - 12.65 = 0.25$ mA

$d_6 = 12.4 - 12.65 = -0.25$ mA

- The average deviation for all measurements is:

$$D = \frac{0.15 + 0.45 + 0.15 + 0.45 + 0.25 + 0.25}{6} = 0.283\text{mA}$$