8. <u>The unit vector normal $\vec{N}$</u> on two vectors $\vec{A}$ and $\vec{B}$

$$\vec{N} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

<u>Example:</u>

Find the unit vector normal on $\vec{A} = 2\vec{i} + 3\vec{j} - \vec{k}$ *and* $\vec{B} = -\vec{j} + 2\vec{k}$?

<u>Sol.</u>

$$\vec{N} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & -1 \\ 0 & -1 & 2 \end{vmatrix} = 5\vec{i} - 4\vec{j} - 2\vec{k}$$

$$|\vec{A} \times \vec{B}| = \sqrt{(5)^2 + (-4)^2 + (-2)^2} = \sqrt{45}$$

$$\vec{N} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|} = \frac{5}{\sqrt{45}}\vec{i} - \frac{4}{\sqrt{45}}\vec{j} - \frac{2}{\sqrt{45}}\vec{k}$$

parallel and orthogonal vectors:

- vectors are orthogonal if:

$\vec{A} \cdot \vec{B} = 0$.

- vectors are parallel if:

(a) $\vec{A} \times \vec{B} = 0$

or

(b) $\vec{A} = c\vec{B}$.

Ex: Determite whether the given vectors are orthogonal, parallel or neither.

$\vec{u} = (-2, 6, -4)$. $\vec{V} = (4, -12, 8)$.

sol

$\vec{V} = 2\vec{u}$, so the vectors are parallel.

$\vec{u} \cdot \vec{V} = (-2 * 4) + (6 * -12) + (-4 * 8)$

$= -8 - 32 - 32$

$= -72$.

So the vectors are not orthogonal.

Example:

Show that $\vec{A} = 2\,\vec{i} + 3\vec{j} + \vec{k}$ and $\vec{B} = 4\,\vec{i} - 2\,\vec{j} - 2\,\vec{k}$ are orthogonal ?

Sol.

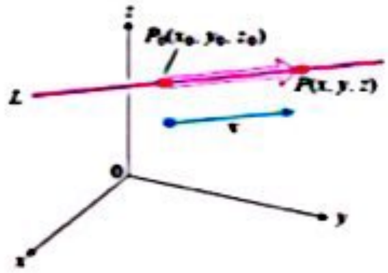
$\vec{A} \cdot \vec{B} = (2 * 4) + (3 * -2) + (1 * -2)$

$= 8 - 6 - 2 = 0 \quad \Rightarrow \quad \vec{A} \perp \vec{B}$

Line and Plane in Space

Parametric Equation of Line

Suppose that **L** is a line in the space passing through the points $\mathbf{p_o}(x_0, y_0, z_0)$ and $\mathbf{P}(x, y, z)$ and parallel to a given vector $\vec{V} = a\,\vec{i} + b\,\vec{j} + c\,\vec{k}$.



$\overrightarrow{p_0\,p} = (x - x_0)\vec{i} + (y - y_0)\vec{j} + (z - z_0)\vec{k}$, is

a vector parallel to $\vec{V}$

$\therefore\ \overrightarrow{p_0\,p} = t\,\vec{V}$, t is scalar parameter.

$$(x - x_0)\vec{i} + (y - y_0)\vec{j} + (z - z_0)\vec{k} = t\left(a\,\vec{i} + b\,\vec{j} + c\,\vec{k}\right)$$

$x = x_0 + at\,,\quad y = y_0 + bt\,,\quad z = z_0 + ct$ **Parametric Equation for Line.**

Example:

Find the parametric equation for line pass through $p_0(-2,0,4)$ and parallel to $\vec{V} = 2\vec{i} + 4\vec{j} - 2\vec{k}$?

Sol.

$(x_0, y_0, z_0) = (-2,0,4)$

$x = x_0 + at \quad \Rightarrow \quad x = -2 + 2t$

$y = y_0 + bt \quad \Rightarrow \quad y = 4t$

$z = z_0 + ct \quad \Rightarrow \quad z = 4 - 2t$

Parametric Equations:

(x_0, y_0, z_0) point

(a, b, c) vector

$X = X_0 + at$

$y = y_0 + bt$

$Z = Z_0 + ct$

point vector

Ex: find the parametric equation for point $(5, 1, 3)$ which is parallel to vector $(i + 4j - 2k)$.

sol

$X = x_0 + at \quad , \quad y = y_0 + bt \quad , \quad Z = Z_0 + ct$

$x = 5 + 1t \quad , \quad y = 1 + 4t \quad , \quad Z = 3 - 2t$

Symmetric Equations for a Line

Consider the parametric form equations for a line **L**:

$$x = x_0 + at \ , \quad y = y_0 + bt \ , \quad z = z_0 + ct$$

If a, b and c are all nonzero, we can solve each equation for **t** to get:

$$\frac{x - x_0}{a} = t$$

$$\frac{y - y_0}{b} = t$$

$$\frac{z - z_0}{c} = t$$

We called these three equations **Symmetric equations** for line **L**, if we set:

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c} = t$$

Example:

Find the symmetric equation for line pass through point (1,-5,6) and is parallel to vector $\vec{v} = -\vec{i} + 2\vec{j} - 3\vec{k}$?

Sol.

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c} = t$$

$$\frac{x - 1}{-1} = \frac{y + 5}{2} = \frac{z - 6}{-3} = t$$

Planes Equation

vectors
↑

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0.$$

Ex: Find the plane equation for point (5,3,4) and vector (1,2,7).
↓ a ↓ b ↓ c

sol

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

$$1(x-5)+2(y-3)+7(z-4)=0$$

$$x-5+2y-6+7z-28=0$$

$$x+2y+7z-39=0$$

$$x+2y+7z=39.$$

Scanned with CamScanner

Vector Equation

$r = r_0 + t v$

r_0 ↓ النقطة المعلومة (x_0, y_0, z_0)

v → متجه موازي (a, b, c).

Ex: Find the vector equation to the line that pass through the point (5, 1, 3) and parallel to vector

$\vec{V} = i + 4j - 2K$.

Sol: $r_0 = 5i + 1j + 3K$.

$\vec{V} = i + 4j - 2K$

$r = r_0 + t v$

$r = 5i + j + 3K + t(i + 4j - 2K)$

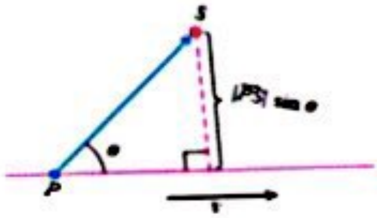
$r = 5i + j + 3K + ti + 4tj - 2tK$.

$r = (5 + t)i + (1 + 4tj) + (3 - 2t)K$.

The Distance from a Point to Line

To find the distance from a point S to a line that passes through a point P and parallel to a vector $\vec{V}$ by:

$$d = \frac{|\overrightarrow{PS} \times \vec{V}|}{|\vec{V}|}$$



Example:

Find the distance from the point S(1,1,5) to the line L:

$x = 1 + t\,, \qquad y = 3 - t\,, \quad z = 2t$

Sol.

From the equation of line that L pass through P(1,3,0) parallel to $\vec{V} = \vec{i} - \vec{j} + 2\vec{k}$

$$\overrightarrow{PS} = (1-1)\vec{i} + (1-3)\vec{j} + (5-0)\vec{k} = -2\vec{j} + 5\vec{k}$$

$$\overrightarrow{PS} \times \vec{V} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -2 & 5 \\ 1 & -1 & 2 \end{vmatrix} = \vec{i} + 5\vec{j} + 2\vec{k}$$

$$d = \frac{|\overrightarrow{PS} \times \vec{V}|}{|\vec{V}|}$$

$$= \frac{\sqrt{(1)^2 + (5)^2 + (2)^2}}{\sqrt{(1)^2 + (-1)^2 + (2)^2}}$$

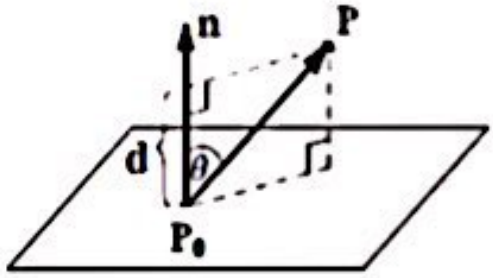
$$= \sqrt{\frac{30}{6}}$$

$$= \sqrt{5}$$

The Distance from a Point to Plane

If **P** is a point on a plane with perpendicular vector $\vec{n}$, then the distance from point P_o to the plane is:

$$d = \frac{|a\,x_o + b\,y_o + c\,z_o + D|}{\sqrt{a^2 + b^2 + c^2}}$$



Example:

Find the distance from the point P (2,4,-5) to the plane $5x - 3y + z = 10$?

Sol.

$$5x - 3y + z - 10 = 0$$

$$d = \frac{|a\,x_o + b\,y_o + c\,z_o + D|}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \frac{|(5 * 2) + (-3 * 4) + (1 * -5) - 10\,|}{\sqrt{(5)^2 + (-3)^2 + (1)^2}}$$

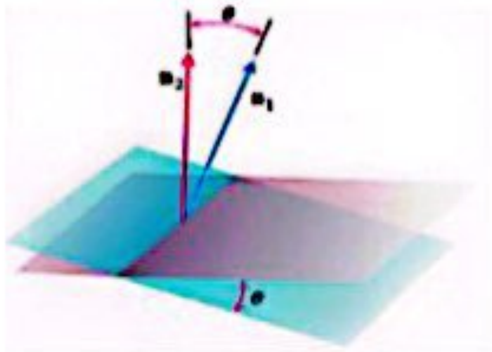
$$= \frac{|-17|}{\sqrt{35}}$$

$$= \frac{17}{\sqrt{35}}$$

Angles Between Planes

The angle between two intersecting planes is defined to be the acute angle determined by their normal vectors.

$$\theta = \cos^{-1}\left(\frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1|\,|\vec{n}_2|}\right)$$



Example:

Find the angle between the planes $3x - 6y - 2z = 15$ *and* $2x + y - 2z = 5$?

Sol.

From the plane equation, the vectors are:

$\vec{n_1} = 3\vec{i} - 6\vec{j} - 2\vec{k}$, $\vec{n_2} = 2\vec{i} + \vec{j} - 2\vec{k}$ *are normal to the plane*

$$\vec{n}_1 \cdot \vec{n}_2 = (3 * 2) + (-6 * 1) + \big(-2 * (-2)\big) = 4$$

$$|\vec{n}_1| = \sqrt{(3)^2 + (-6)^2 + (-2)^2} = \sqrt{49} = 7$$

$$|\vec{n}_2| = \sqrt{(2)^2 + (1)^2 + (-2)^2} = \sqrt{9} = 3$$

$$\theta = \cos^{-1}\left(\frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1|\,|\vec{n}_2|}\right)$$

$$= \cos^{-1}\left(\frac{4}{7 * 3}\right)$$

$$= \cos^{-1}(0.190)$$

$$= 79^{0} \quad or \quad 1.38\, rad$$