Examples: Given $Z_1 = 2+4i$, $Z_2 = 3-i$, find:

(a) $Z_1 + Z_2$

$Z_1 + Z_2 = (2+4i) + (3-i) = 5+3i$

(b) $Z_1 - Z_2 = (2+4i) - (3-i) = 2+4i-3+i = -1+5i$

(c) $Z_2 - Z_1 = (3-i) - (2+4i) = 3-i-2-4i = 1-5i$

Examples: Given $Z_1 = 1-3i$, $Z_2 = -2+5i$, $Z_3 = -3-4i$
find:

(a) $Z_1 Z_2 = (1-3i)(-2+5i)$
$= -2+5i+6i-15i^2 = -2+11i+15 = 13+11i$

(b) $\dfrac{Z_1}{Z_3} = \dfrac{1-3i}{-3-4i} \cdot \dfrac{-3+4i}{-3+4i} = \dfrac{-3+4i+9i-12i^2}{9-12i+12i-16i^2}$

$= \dfrac{9+13i}{9+16} = \dfrac{9+13i}{25}$

(c) $\dfrac{Z_1 Z_2}{Z_1 + Z_2}$ H.w.

Complex equations

If two complex numbers are equal, then their real parts are equal and their imaginary parts are equal. Hence if $a + i\,b = c + i\,d$, then $a = c$ and $b = d$.

Example: Solve the complex equations:

a) $2(x + iy) = 6 - 3\,i$

b) $(1 + 2\,i)(-2 - 3\,i) = a + i\,b$

c) $(2 - 3\,i) = \sqrt{a + i\,b}$ (H.W) **Ans.** $a = -5\,, b = -12$

d) $(x - i\,2y) + (y - i\,3x) = 2 + 3\,i$ **Ans.** $x = -7\,,\ y = 9$

Sol.

a) $2(x + i\,y) = 6 - 3\,i$

$2x + i\,2y = 6 - 3\,i$

$\therefore 2x = 6 \Rightarrow x = 3$

$\therefore 2y = -3 \Rightarrow y = \dfrac{-3}{2}$

b) $(1 + 2\,i)(-2 - 3\,i) = a + i\,b$

$-2 - 3\,i - 4\,i + 6 = a + i\,b$

$4 - 7i = a + i\,b$

$\therefore a = 4 \quad , \quad b = -7$

- Solve the complex equation:

Ex $2(x+iy) = 6-3i$

Sol

$$2x + 2iy = 6 - 3i$$

$$2x = 6 \Rightarrow x = \frac{6}{2} = 3$$

$$2y = -3 \Rightarrow y = \frac{-3}{2}$$

Ex. $(1+2i)(-2-3i) = a+ib$. H.W

- The Modulus and the argoment of complex numbers.

* the Modulus of the complex number $Z = x+iy$ is

$$|Z| = \sqrt{x^2+y^2}$$

* The argoment of the complex number $Z = x+iy$ is:

$$\tan\theta = \frac{y}{x} \Rightarrow \theta = \tan^{-1}\frac{y}{x}$$

θ is the argoment of Z.

* كيفية حساب θ (argoment) في الارباع الاربعة ;

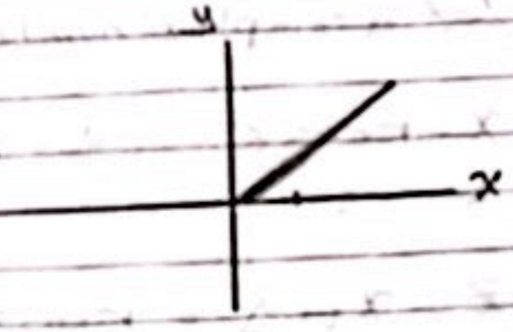
- في الربع الاول تكون نفس القيمة الناتجة من $\theta = \tan^{-1}\frac{y}{x}$
و الربع الرابع ايضا.

- في الربع الثاني تكون $\pi + \tan^{-1}\frac{y}{x}$

- في الربع الثالث تكون $-(\pi - \tan^{-1}\frac{y}{x})$

Ex: Determine the Modulus and argoment of the complex number :

① $1+\sqrt{3}\,i$



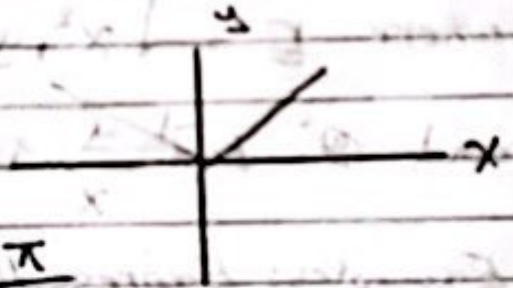
$x=1$, $y=\sqrt{3}$

$|z| = \sqrt{x^2+y^2} = \sqrt{(1)^2+(\sqrt{3})^2}$

$= \sqrt{4} = 2$

$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{\sqrt{3}}{1} = \frac{\pi}{3}$

② $z = +\sqrt{3} + i$



$|z| = \sqrt{(\sqrt{3})^2+(1)^2} = \sqrt{4} = 2$

$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{1}{+\sqrt{3}} = +\frac{\pi}{6}$

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The polar form of complex number:

Let a complex number $Z = x + iy$,

$x = r\cos\theta$, $y = r\sin\theta$.

Hence:

$Z = x + iy = r\cos\theta + i\sin\theta$

$$\boxed{= r(\cos\theta + i\sin\theta)}$$

which is known as the polar form of complex number.

where $r = \sqrt{x^2 + y^2}$ is called the Modulus of Z.

and $\theta = \tan^{-1}\frac{y}{x}$ is called the argument of Z.

Examples: Express the following complex number in polar form:

ⓐ $1 + \sqrt{3}\, i$

$r = \sqrt{x^2 + y^2} = \sqrt{(1)^2 + (\sqrt{3})^2} = \sqrt{4} = 2$

$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{\sqrt{3}}{1} = \frac{\pi}{3}$

$Z = r(\cos\theta + i\sin\theta)$.

$Z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

Example:

Find the value of the complex numbers:

a) $r = 1 \ , \theta = \frac{\pi}{2}$
b) $r = 1 \ , \theta = \pi$
c) $r = 1 \ , \theta = \frac{3\pi}{2}$
d) $r = 1 \ , \theta = 2\pi$

Sol.

a) $r = 1 \ , \theta = \frac{\pi}{2}$

$$z = r(\cos\theta + i\sin\theta)$$

$$z = 1\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

$$z = 1(0 + i \cdot 1)$$

$$z = i$$

c) $r = 1 \ , \theta = \frac{3\pi}{2}$

$$z = r(\cos\theta + i\sin\theta)$$

$$z = 1\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$$

$$z = 1(0 + i \cdot -1)$$

$$z = -i$$

De Moiver's Theorem

For $z = r[\cos\theta + i\sin\theta]$ with power n, we have:

$$\boxed{z^n = r^n[\cos(n\theta) + i\sin(n\theta)]}$$

where n is integer (positive or negative).

Note

To find the value of complex number with power :

1) find r and θ.
2) Use De Moiver's formula $z^n = r^n[\cos(n\theta) + i\sin(n\theta)]$.

Example: Find the value of $a)\ (1+i)^8$, $b)\ (1+i)^{-8}$?

Sol.

$a)\ (1+i)^8$

$$r = \sqrt{x^2 + y^2} = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$$

$$z^n = r^n[\cos(n\theta) + i\sin(n\theta)] \ ,\ n = 8$$

$$(1+i)^8 = \left(\sqrt{2}\right)^8 \left[\cos\left(8 \cdot \frac{\pi}{4}\right) + i\sin\left(8 \cdot \frac{\pi}{4}\right)\right]$$

$$= 16\,[\cos(2\pi) + i\sin(2\pi)]$$

$$= 16[1 + i \cdot 0]$$

$$= 16$$

$b)\ (1+i)^{-8}$

$$r = \sqrt{x^2 + y^2} = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$$

$$z^n = r^n[cos(n\theta) + i\,sin(n\theta)]\ .\ n = -8$$

$$(1+i)^{-8} = \left(\sqrt{2}\right)^{-8}\left[cos\left(-8 \cdot \frac{\pi}{4}\right) + i\,sin\left(-8 \cdot \frac{\pi}{4}\right)\right]$$

$$= \frac{1}{\left(\sqrt{2}\right)^8}[cos(-2\pi) + i\,sin(-2\pi)]$$

$$= \frac{1}{16}[cos(2\pi) - i\,sin(2\pi)]$$

$$= \frac{1}{16}[1 - i \cdot 0]$$

$$= \frac{1}{16}$$