Al-Rasheed University College
Medical Instrumentation Tech. Eng.

Measurements & Medical Transducers

2nd Stage

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Chapter One (Lecture 2)
Error of Measurement

2.1 Types of Errors

- Measurement error is defined as the **difference between** the **measured value** and the **expected value**.
- No measurement can be done perfectly. Errors are always existing. Therefore, it is important to know how different errors have entered into our measurements.
- Errors may come from different source and are usually classified **under four main types**:

1- Gross (Human) errors.

i) Misreading of instruments and observation errors.

ii) Improper choice of instrument, or the range of instrument.

iii) Incorrect adjustment تعديل غير صحيح or forgetting to zero.

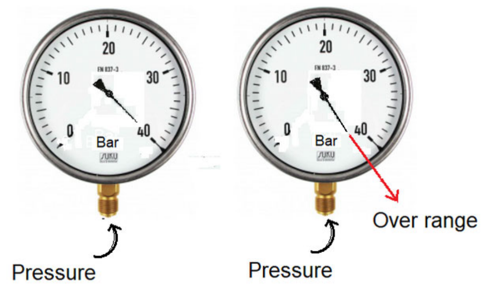
iv) Erroneous calculations, computation mistakes, and estimation errors.

v) Neglect of loading effects.

vi) Proper position for measuring human, i.e., placing EEG probes in wrong positions.

2- Instrumentation (Equipment) Errors

i) Damaged تلف defective عيب equipment such as due to loading effect or worn البالية parts.
ii) Calibration errors.
iii) Component nonlinearities. عملية تصنيع عناصر المواد الكهربائية والألكترونية المكونة لأجهزة القياس تحتوي على شوائب تأثر على أداء تلك العناصر.
iv) Loss during transmission.
v) Proper position of equipment (vertical or horizontal). تأثير الجاذبية



3- Environmental Errors

i) Change in temperature, pressure.
ii) Humidity. الرطوبة
iii) Stray electric and magnetic fields. تأثر جهاز القياس بالمجالات الكهربائية والمغناطيسية الطائشة
iv) Mechanical vibration. الجهاز غير ثابت اثناء عملية القياس
v) Weather variations (day, night, and four seasons). ارتفاع درجات الحرارة في فصل الصيف لأكثر من 40 تؤثر على عمل العديد من أجهزة القياس

4- Random Errors

- Random Errors: are errors occur due to unknown reasons and happen when all other errors are considered. Generally, random errors cannot be fully understood.

- The only ways to reduce this type of errors is by:

▶ Increasing the number of readings of the measured quantity.

▶ Then, we apply mathematical and statistical analysis to determine the best estimate of the measured value.

Calibration (تعيير جهاز القياس) معايرة النظام is the comparison of measurement values delivered by a device under test with those of a calibration standard of known accuracy.

Absolute error $= e = \left| Y - X_i \right|$

error rate, $\%\ Error = \dfrac{\left| Y - X_i \right|}{Y} \times 100\%$

Accuracy, $A = 100\% - \%\ Error$

Average (Mean) $= \bar{X} = \bar{X}_n = \dfrac{\sum_{i=1}^{n} X_i}{n}$

Precision, $P = 1 - \dfrac{\left| X_i - \bar{X} \right|}{\bar{X}}$

Deviation, $d_i = X_i - \bar{X}$ **(+ve OR -ve)**

Average (mean) of deviation, $D = \dfrac{\sum_{i=1}^{n} \left| d_i \right|}{n}$

2.2 Mathematical Equations of Errors

i- Error:

$$\text{error} = |Y - X_i| \tag{1}$$

where:

- Y : the expected value (True value, theoretical).
- Xi : the ith measured value.

Ii- Error rate:

$$\text{error rate} = \frac{|Y - X_i|}{Y} * 100\% \tag{2}$$

$$= \frac{error}{Y} * 100\% \tag{3}$$

iii- Accuracy:

accuracy, A = 100% − error rate (4)

Vi- Average:

$$\bar{X}_n = \frac{X_1 + X_2 + \cdots + X_n}{n} \quad (5)$$

V- Precision:

$$\text{precision} = 1 - \frac{|X_i - \bar{X}_n|}{\bar{X}_n} \quad (6)$$

where:

- ▶ Xi : the ith measurement (reading).
- ▶ X¯n: the average of n readings.

Example 1

Suppose we used an Ohmmeter to measure the value of a resistor. We got the following reading (20.33Ω, 20.18Ω, 20.24Ω, and 20.19Ω). Assume that the expected value is (20Ω).

Sol:

- the error for each one of our readings can be calculated as follows:

$$|20.33 - 20| = 0.33\Omega$$
$$|20.18 - 20| = 0.18\Omega$$
$$|20.24 - 20| = 0.24\Omega$$
$$|20.19 - 20| = 0.19\Omega$$

- error rate for each measurement can be calculated as follows:

$$\text{error rate}_1 = \frac{0.33}{20} * 100\% = 1.65\%$$
$$\text{error rate}_2 = \frac{0.18}{20} * 100\% = 0.9\%$$
$$\text{error rate}_3 = \frac{0.24}{20} * 100\% = 1.2\%$$
$$\text{error rate}_4 = \frac{0.19}{20} * 100\% = 0.95\%$$

- Accuracy for each reading can also be calculated as follows:

$$\begin{aligned} \text{accuracy}_1 &= 100\% - \text{error rate}_1 \\ &= 100\% - 1.65\% = 98.35\% \end{aligned}$$

$$\text{accuracy}_2 = 99.1\%$$

$$\text{accuracy}_3 = 98.8\%$$

$$\text{accuracy}_4 = 99.05\%$$

Example 2: Ten measurements were conducted in the lab and data was recorded in the following table. Calculate the precision of the 4th measurement value.

Sequence	Measurement Value (volts)
1	98
2	102
3	101
4	97
5	100
6	103
7	98
8	106
9	107
10	99

Sol:

- Precision of the 4th measurement value can be calculated using the following:

$$\text{precision}_4 = 1 - \frac{|X_4 - \bar{X}_{10}|}{\bar{X}_{10}}$$

- Average value can be calculated as:

$$\bar{X}_{10} = \frac{98 + 102 + \cdots + 99}{10}$$
$$= 101.1$$

$$\Rightarrow \text{precision}_4 = 1 - \frac{|97 - 101.1|}{101.1}$$
$$= 0.96$$

Example 3:

The expected value of the voltage across a resistor is 80 V. However, the measurement gives a value of 79 V. Calculate the (i)absolute error, (ii) % error, (iii) relative accuracy, and (iv) % of accuracy.

Ans.

(i) Absolute error $e = Y_n - X_n = 80 - 79 = 1$ V

(ii) $\% \text{ Error} = \dfrac{Y_n - X_n}{Y_n} \times 100 = \dfrac{80 - 79}{80} \times 100 = 1.25\%$

(iii) Relative Accuracy

$$A = 1 - \left|\frac{Y_n - X_n}{Y_n}\right| = 1 - \left|\frac{80 - 79}{80}\right|$$

$\therefore \quad A = 1 - 1/80 = 79/80 = 0.9875$

(iv) % of Accuracy $\quad a = 100 \times A = 100 \times 0.9875 = 98.75\%$

or $\quad a = 100\% - \%$ of error $= 100\% - 1.25\% = 98.75\%$

2.3 Random Errors Statistical analysis

Tools are used to determine the best approximation of the measured data and avoiding the random errors in the measurements taken.

1. Arithmetic mean:

$$\bar{x} = \frac{x_1 + x_1 + x_1 + \ldots + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$\bar{x} \rightarrow$ Arithmatic mean

$x_i \rightarrow \mathrm{i^{th}}$ measurement taken

n $\rightarrow$ Number of measurment taken

2. Deviation from the mean

- Each reading of the measurement deviates from the mean by a certain amount. The closer to the mean, is the better approximation to the true value.

$$d_i = x_i - \bar{x}$$

$d_i \rightarrow$ deviation of i^{th} measurement taken from the mean.

$\bar{x} \rightarrow$ Arithmatic mean

$x_i \rightarrow i^{th}$ measurement taken

- The deviation, **d can be either positive or negative**. Note that the algebraic sum (NOT the absolute) of all the deviations equals zero.

2. Average deviation:

- the average deviation indicates the precision of the measuring instruments.

$$D = \frac{|d_1| + |d_1| + \ldots + |d_n|}{n} = \frac{\sum |d|}{n}$$

$d_i \rightarrow$ deviation of i[th] measurement taken from the mean.

$\overline{x} \rightarrow$ Arithmatic mean

$x_i \rightarrow$ i[th] measurement taken

Example 4:
A set of independent current measurements was taken by six observers and recorded as 12.8 mA, 12.2 mA, 12.5 mA, 13.1 mA, 12.9 mA, and 12.4 mA. Calculate (a) the arithmetic mean, (c) the deviation from the mean, (d) The average deviation from the mean.

Ans.

- The arithmetic mean equals:

$$\bar{x} = \frac{12.8 + 12.2 + 12.5 + 13.1 + 12.9 + 12.4}{6} = 12.65\text{mA}$$

- The deviations for each measurement from the mean are:

$d_1 = 12.8 – 12.65 = 0.15$ mA

$d_2 = 12.2 – 12.65 = 0.45$ mA

$d_3 = 12.5 – 12.65 = 0.15$ mA

$d_4 = 13.1 – 12.65 = 0.45$ mA

$d_5 = 12.9 – 12.65 = 0.25$ mA

$d_6 = 12.4 – 12.65 = -0.25$ mA

- The average deviation for all measurements is:

$$D = \frac{0.15 + 0.45 + 0.15 + 0.45 + 0.25 + 0.25}{6} = 0.283\text{mA}$$