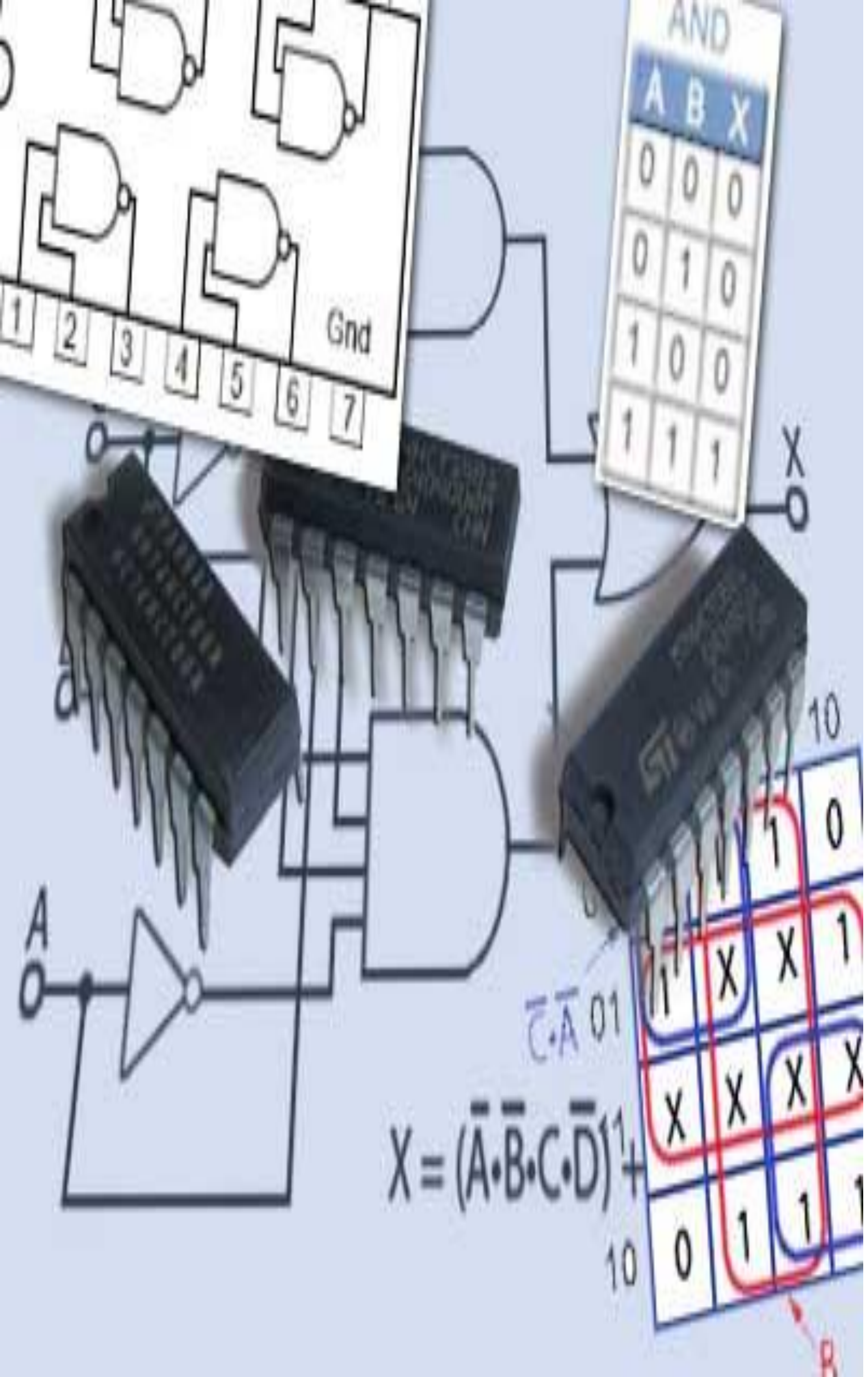

Medical Instrumentations Techniques Engineering
Al-Rasheed University College
Second Level

Digital Techniques
Lecture 08

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LAWS AND THEOREMS OF BOOLEAN ALGEBRA

As in other areas of mathematics, there are certain well-developed rules and laws that must be followed in order to properly apply Boolean algebra. The most important of these are presented in this lecture.

Lecture objectives

At the end of this lecture, the student should be able to:

1- Apply the commutative, associative, and distributive laws of addition and multiplication.

2- Apply twelve basic rules of Boolean algebra.

Laws of Boolean Algebra

The basic laws of Boolean algebra—the commutative laws for addition and multiplication, the associative laws for addition and multiplication, and the distributive law—are the same as in ordinary algebra. Each of the laws is illustrated with two or three variables, but the number of variables is not limited to this.

Commutative laws

The commutative laws of addition or multiplication for two variables are as follows:

$$A + B = B + A$$

$$AB = BA$$

Associative laws

The commutative laws of addition or multiplication for three variables are as follows:

$$A + (B + C) = (A + B) + C$$

$$A(BC) = (AB)C$$

Distributive law

The distributive law for three variables is as follows:

$$A(B + C) = AB + AC$$

Rules of Boolean Algebra

The following table presents the 12 rules of simplifying and manipulating Boolean expressions.

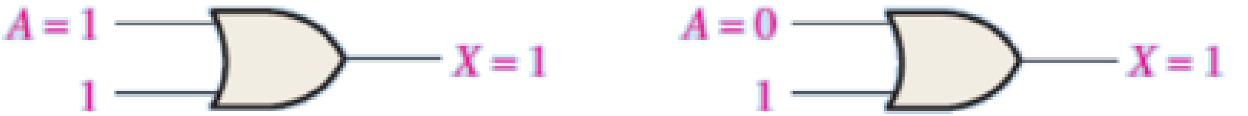
Basic rules of Boolean algebra.

1. $A + 0 = A$	**7.** $A \cdot A = A$
2. $A + 1 = 1$	**8.** $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	**9.** $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	**10.** $A + AB = A$
5. $A + A = A$	**11.** $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	**12.** $(A + B)(A + C) = A + BC$

A, *B*, or *C* can represent a single variable or a combination of variables.

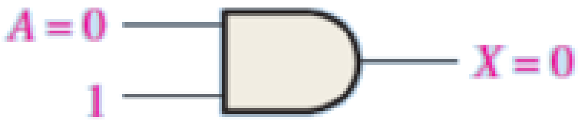
1. $A + 0 = A$

2. $A + 1 = 1$



3. $A \,.\, 0 = 0$

4. $A \cdot 1 = A$



5. $A + A = A$

6. $A + \bar{A} = 1$

7. $A \,.\, A = A$

8. $A \,.\, \bar{A} = 0$

9. $\bar{\bar{A}} = A$

10. $A + AB = A$

$$
\begin{aligned}
A + AB = A \cdot 1 + AB = A(1 + B) \quad & \text{Factoring (distributive law)} \\
= A \cdot 1 \quad & \text{Rule 2: } (1 + B) = 1 \\
= A \quad & \text{Rule 4: } A \cdot 1 = A
\end{aligned}
$$

11. $A + \bar{A}B = A + B$

$$
\begin{aligned}
A + \overline{A}B &= (A + AB) + \overline{A}B && \text{Rule 10: } A = A + AB \\
&= (AA + AB) + \overline{A}B && \text{Rule 7: } A = AA \\
&= AA + AB + A\overline{A} + \overline{A}B && \text{Rule 8: adding } A\overline{A} = 0 \\
&= (A + \overline{A})(A + B) && \text{Factoring} \\
&= 1 \cdot (A + B) && \text{Rule 6: } A + \overline{A} = 1 \\
&= A + B && \text{Rule 4: drop the 1}
\end{aligned}
$$

12. $(A + B)(A + C) = A + BC$

$$
\begin{aligned}
(A + B)(A + C) &= AA + AC + AB + BC && \text{Distributive law} \\
&= A + AC + AB + BC && \text{Rule 7: } AA = A \\
&= A(1 + C) + AB + BC && \text{Factoring (distributive law)} \\
&= A \cdot 1 + AB + BC && \text{Rule 2: } 1 + C = 1 \\
&= A(1 + B) + BC && \text{Factoring (distributive law)} \\
&= A \cdot 1 + BC && \text{Rule 2: } 1 + B = 1 \\
&= A + BC && \text{Rule 4: } A \cdot 1 = A
\end{aligned}
$$

Example: Simplify the following expressions using Boolean algebra rules and laws.

1) $y = A\bar{B}D + A\bar{B}\overline{D}$

2) $z = (\bar{A} + B)(A + B)$

Solution:

1) $y = A\bar{B}D + A\bar{B}\overline{D} = A\bar{B}(D + \overline{D}) = A\bar{B}.1 = A\bar{B}$

2) $z = (\bar{A} + B)(A + B) = \bar{A}\,A + \bar{A}\,B + BA + BB =$

$0 + B(\bar{A} + A) + B = 0 + B + B = B$

Exercise (Lecture 08)

Answer the following:

1- Apply the associative law of addition to the expression:

$A + (B + C + D)$.

2- Apply the distributive law to the expression:

$A(B + C + D)$.

3- Simplify the following expression using Boolean algebra rules and laws.

$y = AB + A(B + C) + B(B + C)$

4- Simplify the following expression using Boolean algebra rules and laws.

$y = [A\bar{B}(C + BD) + \overline{AB}]C$