Al-Rasheed University Collage •
Dept. of Medical Instrument Tech. Eng.
Second Class / Mathematics II•

Inverse Matrices 3*3

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Cofactor Method for Inverses

- Let $\underline{A} = (a_{ij})$ be an nxn matrix

- Recall, the **co-factor** C_{ij} of element a_{ij} is:

$$C_{ij} = (-1)^{i+j} \, |\underline{M}_{ij}|$$

- $\underline{M}_{ij}$ is the (n-1) x (n-1) matrix made by removing the ROW i and COLUMN j of $\underline{A}$

Cofactor Method for Inverses

- Put all **co-factors** in a matrix – called the **matrix of co-factors:**

$$\begin{pmatrix} C_{11} & C_{12} & \cdots & C_{1n} \\ C_{21} & C_{22} & \cdots & C_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ C_{n1} & C_{n2} & \cdots & C_{nn} \end{pmatrix}$$

Cofactor Method for Inverses

- Inverse of $\underline{A}$ is given by:

$$\underline{A}^{-1} = \frac{1}{|\underline{A}|} (\text{matrix of co-factors})^T$$

$$= \frac{1}{|\underline{A}|} \begin{pmatrix} C_{11} & C_{21} & \cdots & C_{n1} \\ C_{12} & C_{22} & \cdots & C_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ C_{1n} & C_{2n} & \cdots & C_{nn} \end{pmatrix}$$

Examples

- Calculate the inverse of $\underline{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\underline{M}_{11} = \begin{pmatrix} d \end{pmatrix}$ $|\underline{M}_{11}| = d$ $C_{11} = d$

Examples

- Calculate the inverse of $\underline{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\underline{M}_{12} = \begin{pmatrix} c \end{pmatrix}$ $|\underline{M}_{12}| = c$ $C_{12} = -c$

Examples

- Calculate the inverse of $\underline{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\underline{M}_{21} = \begin{pmatrix} b \end{pmatrix}$ $\quad |\underline{M}_{21}| = b$ $\quad C_{12} = -b$

Examples

- Calculate the inverse of $\underline{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\underline{M}_{22} = \begin{pmatrix} a \end{pmatrix}$ $|\underline{M}_{22}| = a$ $C_{22} = a$

Examples

- Calculate the inverse of $\underline{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

- Found that:

$$C_{11} = d \qquad C_{12} = -c \qquad C_{21} = -b \qquad C_{22} = a$$

- So,

$$\underline{A}^{-1} = \frac{1}{|\underline{A}|} (\text{matrix of co-factors})^T$$

Examples

- Calculate the inverse of $\underline{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

- Found that:

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- So,

$$\underline{A}^{-1} = \frac{1}{(ad-bc)} \text{(matrix of co-factors)}^T$$

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Examples 3x3 Matrix

- Calculate the inverse of $\underline{B} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{pmatrix}$

- Find the co-factors:

$$\underline{M}_{11} = \begin{pmatrix} 2 & 2 \\ 3 & 4 \end{pmatrix} \qquad |\underline{M}_{11}| = 2 \qquad C_{11} = 2$$

Examples 3x3 Matrix

- Calculate the inverse of $\underline{B} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{pmatrix}$
- Find the co-factors:

$$\underline{M}_{12} = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \qquad |\underline{M}_{12}| = 0 \qquad C_{12} = 0$$

Examples 3x3 Matrix

- Calculate the inverse of $\underline{B} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{pmatrix}$

- Find the co-factors:

$$\underline{M}_{13} = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} \qquad |\underline{M}_{13}| = -1 \qquad C_{13} = -1$$

Examples 3x3 Matrix

- Calculate the inverse of $\underline{B} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{pmatrix}$
- Find the co-factors:

$$\underline{M}_{21} = \begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix} \qquad |\underline{M}_{21}| = 1 \qquad C_{21} = -1$$

Examples 3x3 Matrix

- Calculate the inverse of $\underline{B} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{pmatrix}$
- Find the co-factors:

$\underline{M}_{22} = \begin{pmatrix} 1 & 1 \\ 2 & 4 \end{pmatrix}$ $\quad |\underline{M}_{22}| = 2$ $\quad C_{22} = 2$

Examples 3x3 Matrix

- Calculate the inverse of $\underline{B} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{pmatrix}$
- Find the co-factors:

$$\underline{M}_{23} = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix} \qquad |\underline{M}_{23}| = 1 \qquad C_{23} = -1$$

Examples 3x3 Matrix

- Calculate the inverse of $\underline{B} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{pmatrix}$

- Find the co-factors:

$$\underline{M}_{31} = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} \qquad |\underline{M}_{31}| = 0 \qquad C_{31} = 0$$

Examples 3x3 Matrix

- Calculate the inverse of $\underline{B} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{pmatrix}$
- Find the co-factors:

$\underline{M}_{32} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$ $\quad |\underline{M}_{32}| = 1 \quad C_{32} = -1$

Examples 3x3 Matrix

- Calculate the inverse of $\underline{B} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{pmatrix}$

- First find the co-factors:

$$\underline{M}_{33} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \qquad |\underline{M}_{33}| = 1 \qquad C_{33} = 1$$

Examples 3x3 Matrix

- Calculate the inverse of $\underline{B} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{pmatrix}$

- Next the determinant: use the top row:

$$|\underline{B}| = 1\text{x } |\underline{M}_{11}| \; -1\text{x } |\underline{M}_{12}| + 1\text{x } |\underline{M}_{13}|$$

$$= 2 - 0 + (-1) = 1$$

Examples 3x3 Matrix

- Using the formula,

$$\underline{B}^{-1} = \frac{1}{|\underline{B}|}(\text{matrix of co-factors})^T$$

$$= \frac{1}{1}(\text{matrix of co-factors})^T$$

Examples 3x3 Matrix

- Using the formula,

$$\underline{B}^{-1} = \frac{1}{|\underline{B}|}(\text{matrix of co-factors})^T$$

$$= \frac{1}{1}\begin{pmatrix} 2 & 0 & 1 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}^T$$

Examples 3x3 Matrix

- Using the formula,

$$\underline{B}^{-1} = \frac{1}{|\underline{B}|}(\text{matrix of co-factors})^T$$

$$= \begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ -1 & -1 & 1 \end{pmatrix}$$