



Al-Rasheed University Collage •
Dept. of Medical Instrument Tech. Eng.
Second Class / Mathematics II•

Inverse Matrices 3*3

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Cofactor Method for Inverses

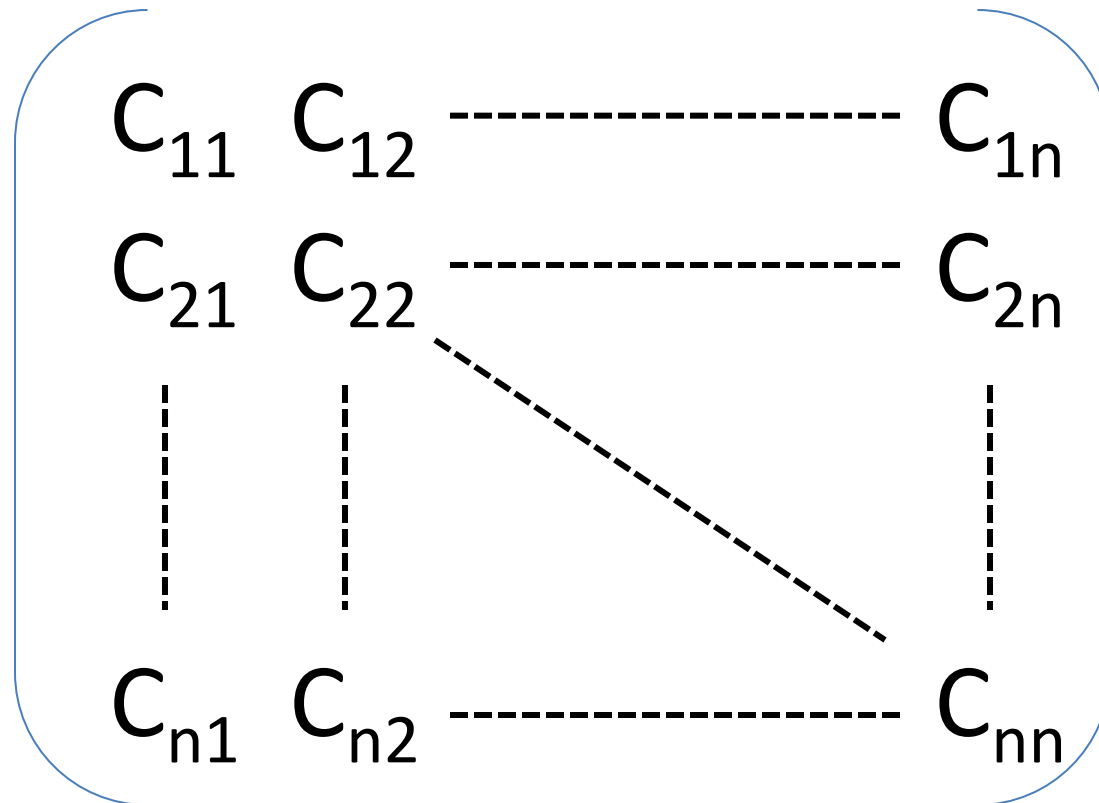
- Let $\underline{A} = (a_{ij})$ be an $n \times n$ matrix
- Recall, the **co-factor** C_{ij} of element a_{ij} is:

$$C_{ij} = (-1)^{i+j} |\underline{M}_{ij}|$$

- \underline{M}_{ij} is the $(n-1) \times (n-1)$ matrix made by removing the ROW i and COLUMN j of \underline{A}

Cofactor Method for Inverses

- Put all **co-factors** in a matrix – called the **matrix of co-factors**:



Cofactor Method for Inverses

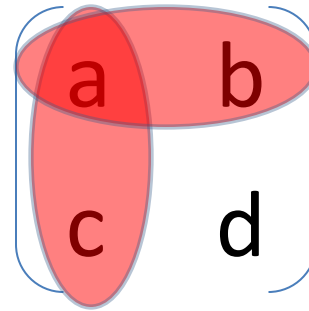
- Inverse of \underline{A} is given by:

$$\underline{A}^{-1} = \frac{1}{|\underline{A}|} (\text{matrix of co-factors})^T$$

$$= \frac{1}{|\underline{A}|} \begin{pmatrix} C_{11} & C_{21} & \cdots & C_{n1} \\ C_{12} & C_{22} & \cdots & C_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ C_{1n} & C_{2n} & \cdots & C_{nn} \end{pmatrix}$$

Examples

- Calculate the inverse of $\underline{A} =$



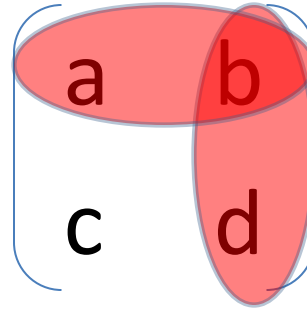
$$\underline{M}_{11} = (d)$$

$$|\underline{M}_{11}| = d$$

$$C_{11} = d$$

Examples

- Calculate the inverse of $\underline{A} =$



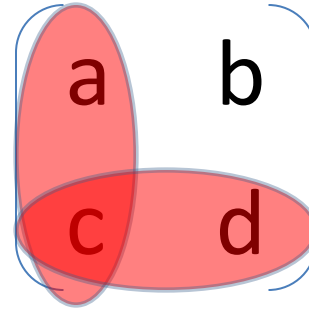
$$\underline{M}_{12} = \left(c \right)$$

$$|\underline{M}_{12}| = c$$

$$C_{12} = -c$$

Examples

- Calculate the inverse of $\underline{A} =$



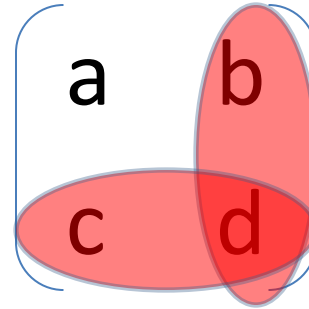
$$\underline{M}_{21} = \begin{pmatrix} b \end{pmatrix}$$

$$|\underline{M}_{21}| = b$$

$$C_{12} = -b$$

Examples

- Calculate the inverse of $\underline{A} =$



$$\underline{M}_{22} = \begin{pmatrix} a \end{pmatrix}$$

$$|\underline{M}_{22}| = a$$

$$C_{22} = a$$

Examples

- Calculate the inverse of $\underline{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

- Found that:

$$C_{11} = d \quad C_{12} = -c \quad C_{21} = -b \quad C_{22} = a$$

- So,

$$\underline{A}^{-1} = \frac{1}{|\underline{A}|} (\text{matrix of co-factors})^T$$

Examples

- Calculate the inverse of $\underline{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

- Found that:

$$C_{11} = d \quad C_{12} = -c \quad C_{21} = -b \quad C_{22} = a$$

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$$\underline{A}^{-1} = \frac{1}{(ad-bc)} (\text{matrix of co-factors})^T$$

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Examples 3x3 Matrix

- Calculate the inverse of $\underline{B} =$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{pmatrix}$$

- Find the co-factors:

$$\underline{M}_{11} = \begin{pmatrix} 2 & 2 \\ 3 & 4 \end{pmatrix}$$

$$|\underline{M}_{11}| = 2$$

$$C_{11} = 2$$

Examples 3x3 Matrix

- Calculate the inverse of $\underline{B} =$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{pmatrix}$$

- Find the co-factors:

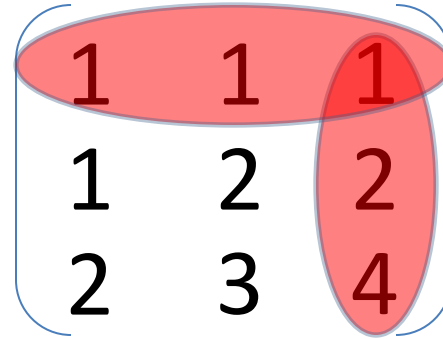
$$\underline{M}_{12} = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$$

$$|\underline{M}_{12}| = 0$$

$$C_{12} = 0$$

Examples 3x3 Matrix

- Calculate the inverse of $\underline{B} =$


$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{pmatrix}$$

- Find the co-factors:

$$\underline{M}_{13} = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$$

$$|\underline{M}_{13}| = -1$$

$$C_{13} = -1$$

Examples 3x3 Matrix

- Calculate the inverse of $\underline{B} =$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{pmatrix}$$

- Find the co-factors:

$$\underline{M}_{21} = \begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix}$$

$$|\underline{M}_{21}| = 1$$

$$C_{21} = -1$$

Examples 3x3 Matrix

- Calculate the inverse of $\underline{B} =$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{pmatrix}$$

- Find the co-factors:

$$\underline{M}_{22} = \begin{pmatrix} 1 & 1 \\ 2 & 4 \end{pmatrix}$$

$$|\underline{M}_{22}| = 2$$

$$C_{22} = 2$$

Examples 3x3 Matrix

- Calculate the inverse of $\underline{B} =$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{pmatrix}$$

- Find the co-factors:

$$\underline{M}_{23} = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$$

$$|\underline{M}_{23}| = 1$$

$$C_{23} = -1$$

Examples 3x3 Matrix

- Calculate the inverse of $\underline{B} =$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{pmatrix}$$

- Find the co-factors:

$$\underline{M}_{31} = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$$

$$|\underline{M}_{31}| = 0$$

$$C_{31} = 0$$

Examples 3x3 Matrix

- Calculate the inverse of $\underline{B} =$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{pmatrix}$$

- Find the co-factors:

$$\underline{M}_{32} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$$

$$|\underline{M}_{32}| = 1$$

$$C_{32} = -1$$

Examples 3x3 Matrix

- Calculate the inverse of $\underline{B} =$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{pmatrix}$$

- First find the co-factors:

$$\underline{M}_{33} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$$

$$|\underline{M}_{33}| = 1$$

$$C_{33} = 1$$

Examples 3x3 Matrix

- Calculate the inverse of $\underline{B} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{pmatrix}$
- Next the determinant: use the top row:

$$\begin{aligned} |\underline{B}| &= 1x |\underline{M}_{11}| - 1x |\underline{M}_{12}| + 1x |\underline{M}_{13}| \\ &= 2 - 0 + (-1) = 1 \end{aligned}$$

Examples 3x3 Matrix

- Using the formula,

$$\underline{\mathbf{B}}^{-1} = \frac{1}{|\underline{\mathbf{B}}|} (\text{matrix of co-factors})^T$$

$$= \frac{1}{1} (\text{matrix of co-factors})^T$$

Examples 3x3 Matrix

- Using the formula,

$$\underline{\underline{B}}^{-1} = \frac{1}{|\underline{\underline{B}}|} (\text{matrix of co-factors})^T$$

$$= \frac{1}{1} \begin{pmatrix} 2 & 0 & 1 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}^T$$

Examples 3x3 Matrix

- Using the formula,

$$\underline{\underline{B}}^{-1} = \frac{1}{|\underline{\underline{B}}|} (\text{matrix of co-factors})^T$$

$$= \begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ -1 & -1 & 1 \end{pmatrix}$$