Al-Rasheed University Collage

Dept. of Medical Instrument Tech. Eng.

First Class / Mathematics

MATRICES

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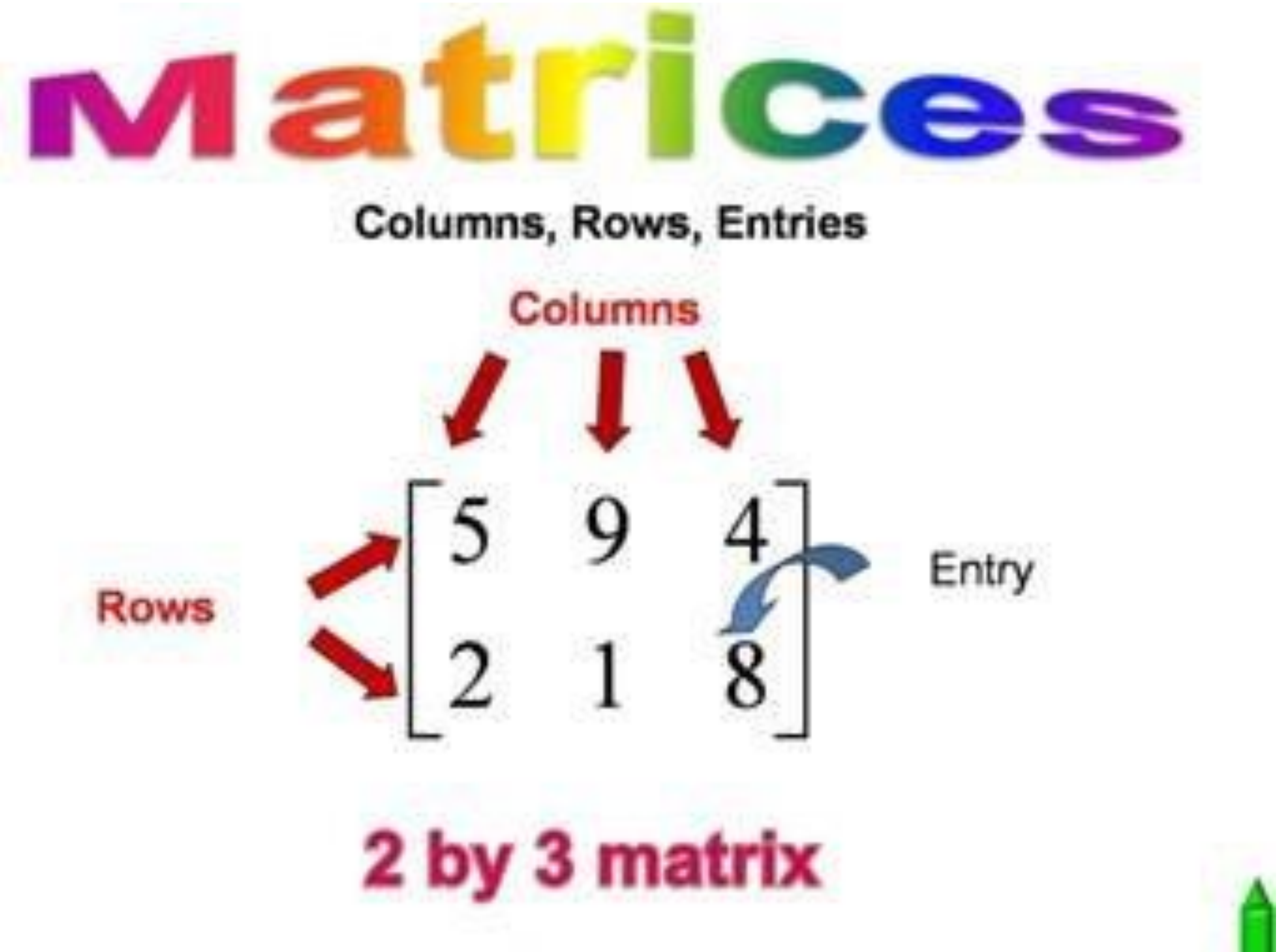
Definition of matrix

- **A matrix is a rectangular arrangement of numbers in rows (سطور) and columns (اعمدة) . Rows run horizontally and columns run vertically.**

- **The dimensions of a matrix are stated "*m* x *n*" where '*m*' is the number of rows and '*n*' is the number of columns.**


Matrices
Columns, Rows, Entries
Columns
Rows
5 9 4
2 1 8
Entry
2 by 3 matrix

SPECIAL MATRICES

(بعض المصفوفات الخاصة)

Name	Description	Example
Row Matrix	only 1 row	$\begin{bmatrix} 3 & -2 & 4 \end{bmatrix}$
Column Matrix	only 1 column	$\begin{bmatrix} 1 \\ 3 \\ 7 \end{bmatrix}$
Square matrix	same # of rows and columns	$\begin{bmatrix} 7 & -3 \\ 8 & 4 \end{bmatrix}$
Zero Matrix	all entries are zeros	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

1. Equal Matrices (تساوي المصفوفات)

Two matrices are considered equal if :

a. They have the same number of rows and columns (the same dimensions)

b. All their corresponding elements are exactly the same.

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \neq B = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$$

- Are these equal?

$$\begin{bmatrix} 5 & 0 \\ -\frac{4}{4} & \frac{3}{4} \end{bmatrix} \quad \begin{bmatrix} 5 & 0 \\ -1 & 0.75 \end{bmatrix}$$

2. Addition and Subtraction Matrices

(جمع وطرح المصفوفات)

- You can add or subtract matrices **if** they have the same dimensions (**same number of rows and columns**).

n1=n2 and m1=m2

- Then, you add (or subtract) the corresponding numbers (numbers in the same positions).

- Examples:

$$\begin{bmatrix} 3 \\ -4 \\ 7 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

➢ **Possible**

$$\begin{bmatrix} 2 & 5 \\ -4 & 7 \end{bmatrix} - \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

➢ **Not Possible**

Example 1.1:

$$\begin{bmatrix} 2 & -4 \\ 5 & 0 \\ 1 & -3 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ -2 & 1 \\ 3 & -3 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 7 & -1 \\ -2 & 0 \end{bmatrix}$$

Example 1.2:

$$\begin{bmatrix} 4 & -3 & 1 \\ 0 & 5 & -2 \\ 5 & -6 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 3 \\ 6 & -7 & 9 \\ 0 & -4 & 8 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 4 \\ 6 & -2 & 7 \\ 5 & -10 & 8 \end{bmatrix}$$

Example 1.3:

$$\begin{bmatrix} 4 & -3 & 1 \\ 0 & 5 & -2 \\ 5 & -6 & 0 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 3 \\ 6 & -7 & 9 \\ 0 & -4 & 8 \end{bmatrix} = \begin{bmatrix} 4-(-1) & -3-2 & 1-3 \\ 0-6 & 5-(-7) & -2-9 \\ 5-0 & -6-(-4) & 0-8 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -5 & -2 \\ -6 & 12 & -11 \\ 5 & -2 & -8 \end{bmatrix}$$

YOUR TURN!

- Perform the indicated operation, if possible.

a. $\begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ -9 \\ 8 \end{bmatrix}$

b. $\begin{bmatrix} 3 & 3 \\ 9 & -5 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ -3 & 5 \end{bmatrix}$

c. $\begin{bmatrix} 9 & -7 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ 4 & -8 \end{bmatrix}$

3. Scalar Multiplication
(الضرب القياسي)

- **Scalar** – a regular number
- **Scalar multiplication** – multiplying a matrix by a scalar
  - Just multiply each entry by the scalar!

Example:

$$3\begin{bmatrix} -2 & 0 \\ 4 & -7 \end{bmatrix}$$

Example 1.4:

$$4\begin{bmatrix} 2 & -4 \\ 5 & 0 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 8 & -16 \\ 20 & 0 \\ 4 & -12 \end{bmatrix}$$

Your Turn!

- Perform the indicated operation(s).

$$-1\begin{bmatrix} 4 & -7 \\ 3 & 3 \\ 2 & -9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 9 & -8 \\ 1 & -4 \end{bmatrix}$$

- Perform the indicated operation(s), if possible.

$$-2\begin{bmatrix} 1 & -2 \\ 0 & 3 \\ -4 & 5 \end{bmatrix} + \begin{bmatrix} -4 & 5 \\ 6 & -8 \\ -2 & 6 \end{bmatrix}$$

4. Matrix Multiplication (ضرب المصفوفات)

- **Matrix Multiplication is NOT Commutative! Order matters!**

(ضرب المصفوفات ليس تبادلي)

- **You can multiply matrices <u>only</u> if the number of columns in the first matrix equals the number of rows in the second matrix.**

الشرط : (ألاعمدة =السطور)

2 columns

$$\begin{bmatrix} 2 & 3 \\ -5 & 6 \\ 9 & -7 \end{bmatrix} \bullet \begin{bmatrix} 1 & -2 & 0 \\ 3 & 4 & -5 \end{bmatrix}$$

2 rows

Example 1.5

- **Take the numbers in the first row of matrix #1. Multiply each number by its corresponding number in the first column of matrix #2. Total these products.**

$$\begin{bmatrix} 2 & 3 \\ -5 & 6 \\ 9 & -7 \end{bmatrix} \bullet \begin{bmatrix} 1 & -2 & 0 \\ 3 & 4 & -5 \end{bmatrix}$$

The result, 11, goes in row 1, column 1 of the answer. Repeat with row 1, column 2; row 1 column 3; row 2, column 1; ...

(2*1)+(3*3)=11

- **Notice the dimensions of the matrices and their product.**

$$\begin{bmatrix} 2 & 3 \\ -5 & 6 \\ 9 & -7 \end{bmatrix} \bullet \begin{bmatrix} 1 & -2 & 0 \\ 3 & 4 & -5 \end{bmatrix} = \begin{bmatrix} 11 & 8 & -15 \\ 13 & 34 & -30 \\ -12 & -46 & 35 \end{bmatrix}$$

3 x 2   2 x 3   3 x 3

Example 1.6:

Given A = $\begin{bmatrix} 3 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix}$ **B=** $\begin{bmatrix} 1 & 6 \\ 3 & -5 \\ -2 & 4 \end{bmatrix}$

Find AB if it is defined: $\begin{bmatrix} 3 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 3 & -5 \\ -2 & 4 \end{bmatrix}$

$$\begin{bmatrix} 3 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 3 & -5 \\ -2 & 4 \end{bmatrix}$$

Since A is a 2 x 3 matrix and B is a 3 x 2 matrix, **AB** will be a 2 x 2 matrix.

1. Multiply first row of A by first column of B :

3(1) + 1(3) +(-1)(-2)=8

2. First row of A times **second** column of B:

3(6)+1(-5)+ (-1)(4)= 9

3. Proceeding as above the final result is

$$= \begin{bmatrix} 8 & 9 \\ -4 & 24 \end{bmatrix}$$

Example 1.7:

$$\begin{bmatrix} 10 & 5 & 8 & 3 \end{bmatrix} \begin{bmatrix} 12,500 \\ 11,800 \\ 15,900 \\ 25,300 \end{bmatrix} = \begin{bmatrix} 10(12,500) + 5(11,800) + 8(15,900) + 3(25,300) \end{bmatrix} = \begin{bmatrix} 387,100 \end{bmatrix}$$

5. Transpose of A Matrix

If the rows and columns of a matrix are interchanged, then the new matrix so formed is called the transpose of the original matrix. If A is the original matrix, its transpose is denoted by A^T. For example:

If $A = \begin{bmatrix} 4 & 6 \\ 7 & 9 \\ 2 & 5 \end{bmatrix}_{3\times 2}$, then $A^T = \begin{bmatrix} 4 & 7 & 2 \\ 6 & 9 & 5 \end{bmatrix}_{2\times 3}$