Al-Rasheed University Collage

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First Class / Mathematics

Method of integration (u-substitution)-2

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EXAMPLE 8.2.1 Evaluate $\int \sin^5 x\,dx$. Rewrite the function:

$$\int \sin^5 x\,dx = \int \sin x \sin^4 x\,dx = \int \sin x(\sin^2 x)^2\,dx = \int \sin x(1-\cos^2 x)^2\,dx.$$

Now use $u = \cos x$, $\frac{du}{dx} = -\sin(x) \rightarrow$ $dx = \frac{du}{-\sin(x)}$

$$\int \sin(x)(1-u^2)^2 \frac{du}{-\sin(x)} = \int -(1-u^2)^2\,du = -\int (1-u^2)^2\,du$$

$$-\int (1-2u^2+u^4)\,du$$

$$= -u + \frac{2}{3}u^3 - \frac{1}{5}u^5 + C$$

$$= -\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C.$$

EXAMPLE 8.2.2 Evaluate $\int \sin^6 x\,dx$. Use $\sin^2 x = (1-\cos(2x))/2$ to rewrite the function:

$$\int \sin^6 x\,dx = \int (\sin^2 x)^3\,dx = \int \frac{(1-\cos 2x)^3}{8}\,dx = \frac{1}{8}\int (1-\cos(2x))^2(1-\cos(2x))dx$$

$$\int (1-2\cos(2x)+\cos^2(2x))(1-\cos(2x))dx = \frac{1}{8}\int 1-3\cos 2x+3\cos^2 2x-\cos^3 2x\,dx.$$

Now we have four integrals to evaluate:

$$\int 1\,dx = x$$

and

$$\int -3\cos 2x\,dx = -\frac{3}{2}\sin 2x$$

$\cos kx$	$\frac{1}{k}\sin kx + c$
$\sin kx$	$-\frac{1}{k}\cos kx + c$

are easy. The $\cos^3 2x$ integral is like the previous example:

$$\int -\cos^3 2x\,dx = \int -\cos 2x \cos^2 2x\,dx$$
$$= \int -\cos 2x(1-\sin^2 2x)\,dx$$

$$\text{let } u = \sin 2x \quad \frac{du}{dx} = 2\cos 2x \to dx = \frac{du}{2\cos 2x} \to \int -\cos 2x\,(1-u^2)\frac{du}{2\cos 2x} = -\frac{1}{2}\int (1-u^2)\,du$$

$$= -\frac{1}{2}\left(u - \frac{u^3}{3}\right)$$
$$= -\frac{1}{2}\left(\sin 2x - \frac{\sin^3 2x}{3}\right).$$

And finally we use another trigonometric identity,

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\int 3\cos^2 2x\,dx = 3\int \frac{1+\cos 4x}{2}\,dx = \frac{3}{2}\left(x + \frac{\sin 4x}{4}\right).$$

So at long last we get

$$\int \sin^6 x\,dx = \frac{x}{8} - \frac{3}{16}\sin 2x - \frac{1}{16}\left(\sin 2x - \frac{\sin^3 2x}{3}\right) + \frac{3}{16}\left(x + \frac{\sin 4x}{4}\right) + C.$$

EXAMPLE 8.2.3 Evaluate $\int \sin^2 x \cos^2 x \, dx$. Use the formulas $\sin^2 x = (1-\cos(2x))/2$ and $\cos^2 x = (1+\cos(2x))/2$ to get:

$\cos kx$	$\frac{1}{k}\sin kx + c$

$$\int \sin^2 x \cos^2 x \, dx = \int \frac{1-\cos(2x)}{2} \cdot \frac{1+\cos(2x)}{2} \, dx.$$

$$\frac{1}{4}\int [1 - \cos^2(2x)]dx = \frac{1}{4}\int \{1 - \left[\frac{1+\cos(4x)}{2}\right]\}dx$$

$$\frac{1}{4}\int \{\frac{1}{2} - \frac{\cos(4x)}{2}\}dx = \frac{1}{4}\{\int \frac{1}{2}dx - \int \frac{\cos(4x)}{2}dx\}$$

$$\frac{1}{4}\left\{\frac{1}{2}x - \frac{\sin(4x)}{8} + c\right\} = \frac{1}{8}x - \frac{\sin(4x)}{32} + h$$

Find the antiderivatives.

1. $\int \sin^2 x \, dx \Rightarrow$

2. $\int \sin^3 x \, dx \Rightarrow$

3. $\int \sin^4 x \, dx \Rightarrow$

4. $\int \cos^2 x \sin^3 x \, dx \Rightarrow$

5. $\int \cos^3 x \, dx \Rightarrow$

6. $\int \sin^2 x \cos^2 x \, dx \Rightarrow$

7. $\int \cos^3 x \sin^2 x \, dx \Rightarrow$

8. $\int \sin x (\cos x)^{3/2} \, dx \Rightarrow$

9. $\int \sec^2 x \csc^2 x \, dx \Rightarrow$

10. $\int \tan^3 x \sec x \, dx \Rightarrow$