Al-Rasheed University Collage

Dept. of Medical Instrument Tech. Eng.

First Class / Mathematics

Rules of integral

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The integral of $k\,f(x)$ where k is a constant

A constant factor in an integral can be moved outside the integral sign as follows:

Key Point 1

$$\int k\,f(x)\,dx = k\int f(x)\,dx$$

Find the indefinite integral of $11x^2$: that is, find $\int 11x^2\,dx$

Solution

$$\int 11x^2\,dx = 11\int x^2\,dx = 11\left(\frac{x^3}{3}+c\right) = \frac{11x^3}{3}+K$$ where K is a constant.

Find the indefinite integral of $-5\cos x$; that is, find $\int -5\cos x\,dx$

Solution

$$\int -5\cos x\,dx = -5\int \cos x\,dx = -5\left(\sin x + c\right) = -5\sin x + K$$ where K is a constant.

The integral of $f(x) + g(x)$ and of $f(x) - g(x)$

When we wish to integrate the sum or difference of two functions, we integrate each term separately as follows:

Key Point 2

$$\int [\, f(x) + g(x) \,]\, dx = \int f(x)\, dx + \int g(x)\, dx$$

$$\int [\, f(x) - g(x) \,]\, dx = \int f(x)\, dx - \int g(x)\, dx$$

Find $\int (x^3 + \sin x)\, dx$

Solution

$$\int (x^3 + \sin x)\, dx = \int x^3\, dx + \int \sin x\, dx = \frac{1}{4}x^4 - \cos x + c$$

Note that only a single constant of integration is needed.

Find $\int (3t^4 + \sqrt{t})\, dt$

The hyperbolic sine and cosine functions, $\sinh x$ and $\cosh x$, are defined as follows:

$$\sinh x = \frac{\mathrm{e}^x - \mathrm{e}^{-x}}{2} \qquad \cosh x = \frac{\mathrm{e}^x + \mathrm{e}^{-x}}{2}$$

Note that they are combinations of the exponential functions e^x and e^{-x}.
Find the indefinite integrals of $\sinh x$ and $\cosh x$.

$$\int \sinh x \, dx = \int \left(\frac{\mathrm{e}^x - \mathrm{e}^{-x}}{2} \right) dx =$$

Answer

$$\int \sinh x \, dx = \frac{1}{2} \int \mathrm{e}^x \, dx - \frac{1}{2} \int \mathrm{e}^{-x} \, dx = \frac{1}{2}\mathrm{e}^x + \frac{1}{2}\mathrm{e}^{-x} + c = \frac{1}{2}\left(\mathrm{e}^x + \mathrm{e}^{-x}\right) + c = \cosh x + c.$$

$$\int \cosh x \, dx = \int \left(\frac{\mathrm{e}^x + \mathrm{e}^{-x}}{2} \right) dx =$$

Exercises

1. Find $\int (2x - \mathrm{e}^x)\, dx$
2. Find $\int 3\mathrm{e}^{2x}\, dx$
3. Find $\int \frac{1}{3}(x + \cos 2x)\, dx$
4. Find $\int 7x^{-2}\, dx$
5. Find $\int (x+3)^2\, dx$, (be careful!)