Al-Rasheed University Collage
Dept. of Medical Instrument Tech. Eng.
First Class / Mathematics

Indefinite integral

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1. Integration as differentiation in reverse

Suppose we differentiate the function $y = x^2$. We obtain $\frac{dy}{dx} = 2x$. Integration reverses this process and we say that the integral of $2x$ is x^2. Pictorially we can regard this as shown in Figure 1:

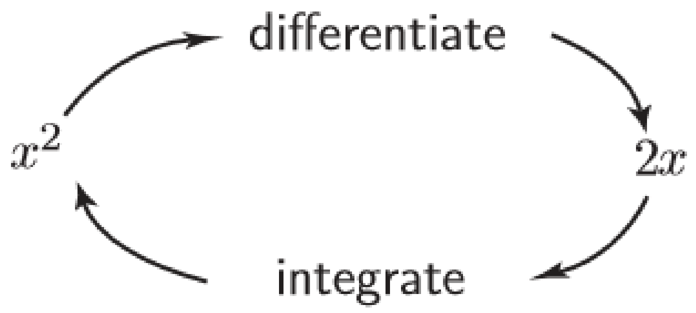
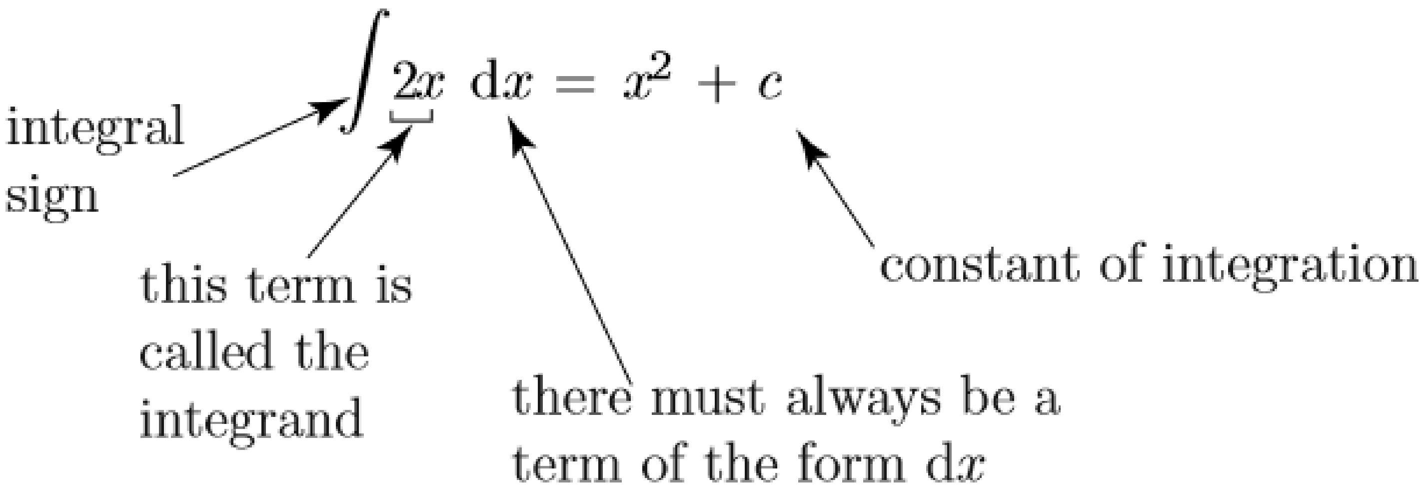


Figure 1

The symbol for integration, $\int$, is known as an **integral sign**. To integrate $2x$ we write



Exercises

1. (a) Write down the derivatives of each of: x^3, $x^3 + 17$, $x^3 - 21$

   (b) Deduce that $\int 3x^2\,dx = x^3 + c$.

2. Explain why, when finding an indefinite integral, a constant of integration is always needed.

Answers

1. (a) $3x^2$, $3x^2$, $3x^2$ (b) Whatever the constant, it is zero when differentiated.

2. Any constant will disappear (i.e. become zero) when differentiated so one must be reintroduced to reverse the process.

Table 1: Integrals of Common Functions

function $f(x)$	indefinite integral $\int f(x)\,dx$
constant, k	$kx + c$
x	$\frac{1}{2}x^2 + c$
x^2	$\frac{1}{3}x^3 + c$
x^n	$\dfrac{x^{n+1}}{n+1} + c, \quad n \neq -1$
x^{-1} (or $\dfrac{1}{x}$)	$\ln\lvert x\rvert + c$
$\cos x$	$\sin x + c$
$\sin x$	$-\cos x + c$
$\cos kx$	$\dfrac{1}{k}\sin kx + c$
$\sin kx$	$-\dfrac{1}{k}\cos kx + c$
$\tan kx$	$\dfrac{1}{k}\ln\lvert\sec kx\rvert$+c
e^{x}	$e^{x} + c$
e^{-x}	$-e^{-x} + c$
e^{kx}	$\dfrac{1}{k}e^{kx} + c$

Example 1

Use Table 1 to find the indefinite integral of x^7: that is, find $\int x^7\,dx$

Solution

From Table 1 note that $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + c.$ In words, this states that to integrate a power of x, increase the power by 1, and then divide the result by the new power. With $n = 7$ we find

$$\int x^7\,dx = \frac{1}{8}x^8 + c$$

Example 2

Find the indefinite integral of $\cos 5x$: that is, find $\int \cos 5x \, dx$

Solution

From Table 1 note that $\displaystyle\int \cos kx \, dx = \frac{\sin kx}{k} + c$

With $k = 5$ we find $\displaystyle\int \cos 5x \, dx = \frac{1}{5} \sin 5x + c$

Example 3

Find $\int \cos 5t \, dt$

Solution

We integrated $\cos 5x$ in the previous example. Now the independent variable is t, so simply use Table 1 and replace every x with a t. With $k = 5$ we find

$$\int \cos 5t \, dt = \frac{1}{5} \sin 5t + c$$

It follows immediately that, for example,

$$\int \cos 5\omega \, d\omega = \frac{1}{5} \sin 5\omega + c, \qquad \int \cos 5u \, du = \frac{1}{5} \sin 5u + c \quad \text{and so on.}$$

Example 4

Find the indefinite integral of $\frac{1}{x}$: that is, find $\int \frac{1}{x}\, dx$

Solution

This integral deserves special mention. You may be tempted to try to write the integrand as x^{-1} and use the fourth row of Table 1. However, the formula $\int x^n\, dx = \frac{x^{n+1}}{n+1} + c$ is not valid when $n = -1$ as Table 1 makes clear. This is because we can never divide by zero. Look to the fifth entry of Table 1 and you will see $\int x^{-1}\, dx = \ln|x| + c$.

Example 5

Find $\int 12\,dx$ and $\int 12\,dt$

Solution

In this Example we are integrating a constant, 12. Using Table 1 we find

$$\int 12\,dx = 12x + c \qquad \text{Similarly } \int 12\,dt = 12t + c.$$

Exercises

1. Integrate each of the following functions with respect to x:
   (a) x^9, (b) $x^{1/2}$, (c) x^{-3}, (d) $1/x^4$, (e) 4, (f) $\sqrt{x}$, (g) e^{4x}

2. Find (a) $\int t^2\,dt$, (b) $\int 6\,dt$, (c) $\int \sin 3t\,dt$, (d) $\int \mathrm{e}^{7t}\,dt$.